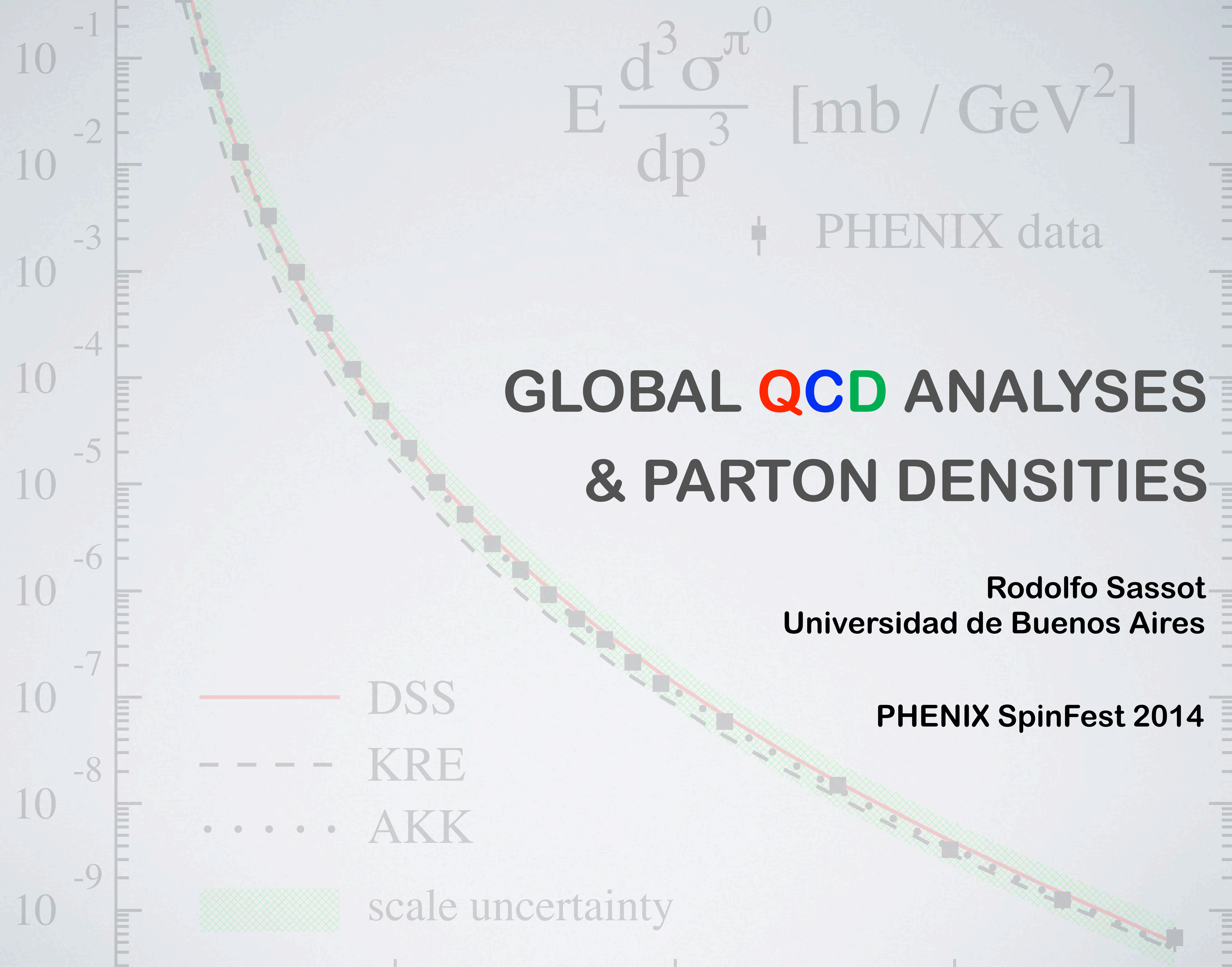


GLOBAL QCD ANALYSES & PARTON DENSITIES

Rodolfo Sassot
Universidad de Buenos Aires

PHENIX SpinFest 2014

Second Lecture: Fragmentation Functions

2.1 FFs and jets:

2.2 SIA, SIDIS and pp:

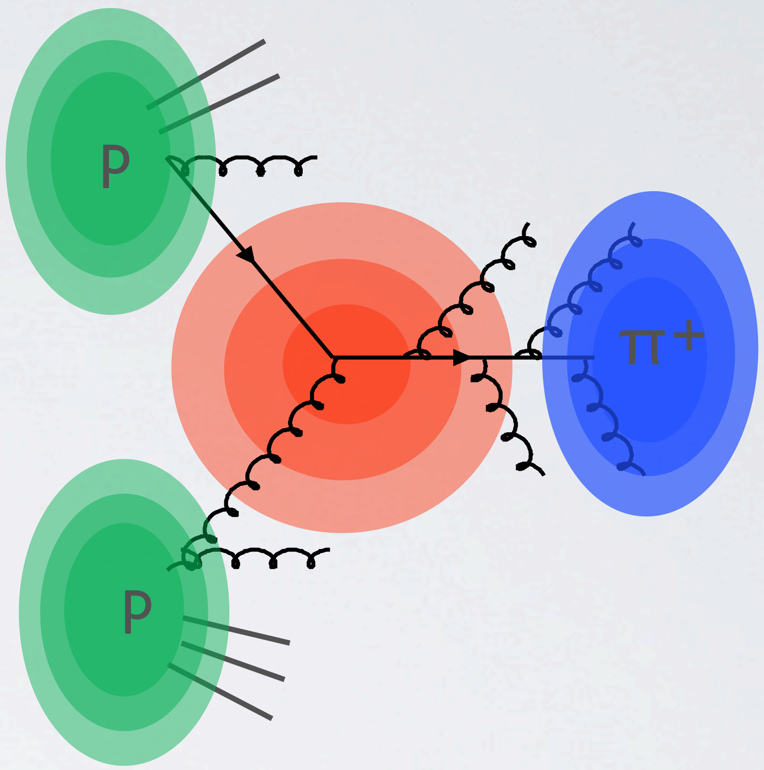
2.3 Global FFs fits: DSS

2.4 Uncertainties: Lagrange

2.5 Uncertainties: Hessian

2.6 DSS-II

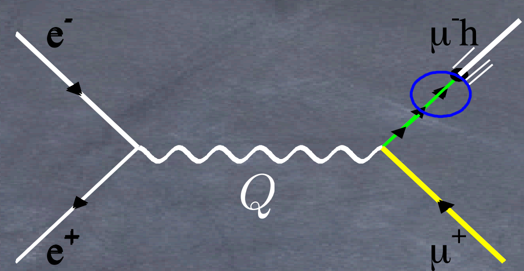
2.7 Outlook



$$\sigma = f_i \otimes f_j \otimes \sigma_{ijk} \otimes D_k^{\pi}$$

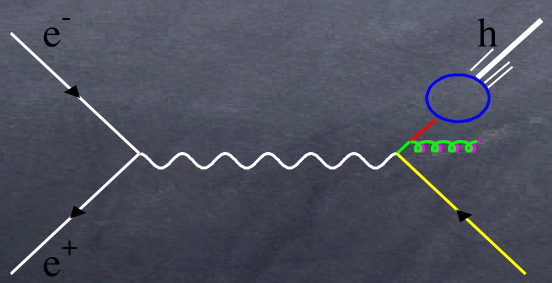
2.1 FFs and jets

Looking for FFs:



$$z \equiv \frac{E_h}{E_q} = \frac{2E_h}{Q}$$

$$\frac{d\sigma}{dz}(e^+e^- \to h\,X) = \sum_q \sigma(e^+e^- \to q\bar{q})\left[D_q^h(z) + D_{\bar{q}}^h(z)\right]$$

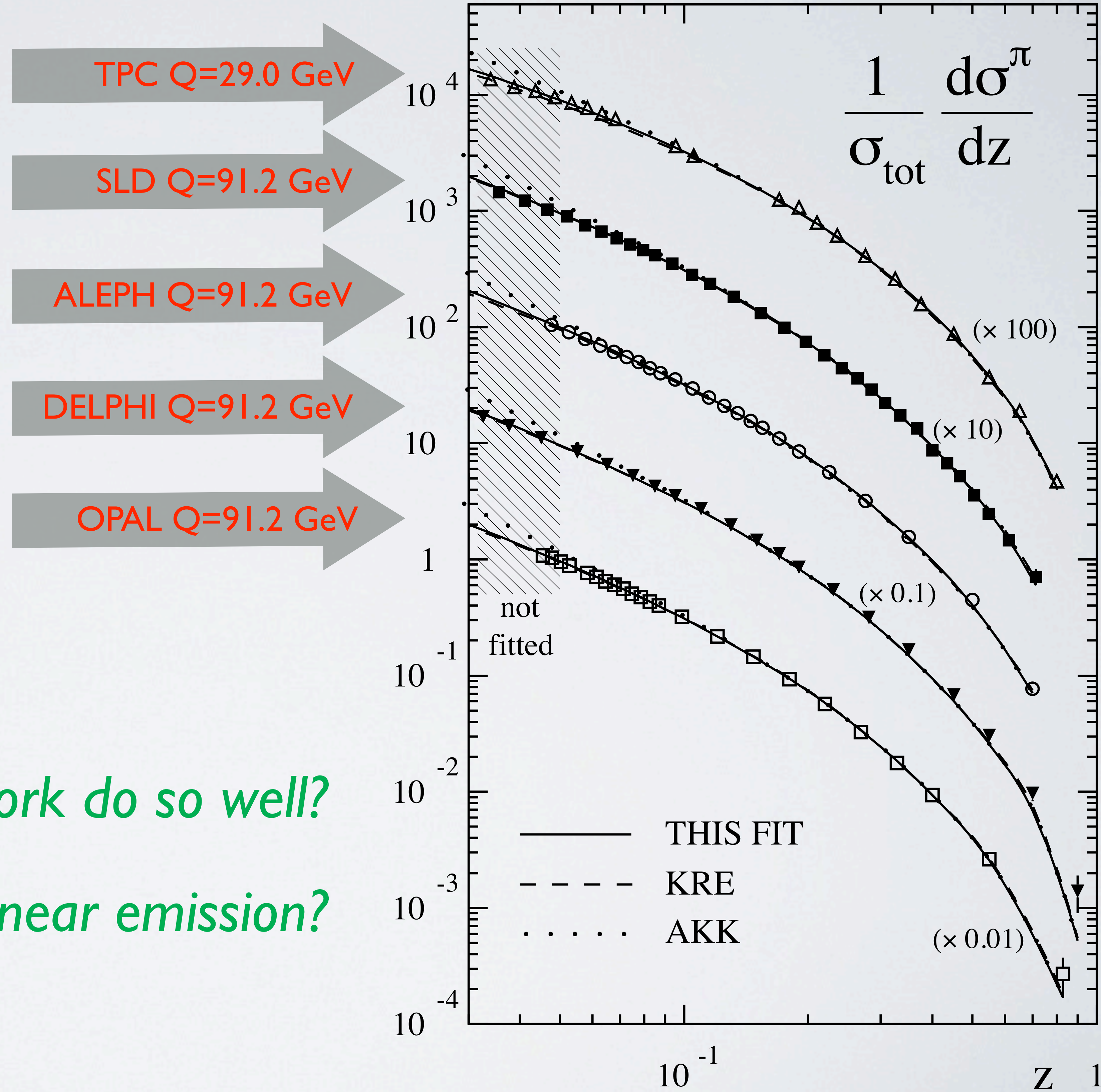


$$D_q^h(z) \longrightarrow D_q^h(z, Q^2)$$

- describe collinear transition of a parton i into hadron h
- universal (process independent)
- known scale (energy) dependence
- unavoidable if identified hadronic final state

same theoretical footing as PDFs

2.1 FFs and jets

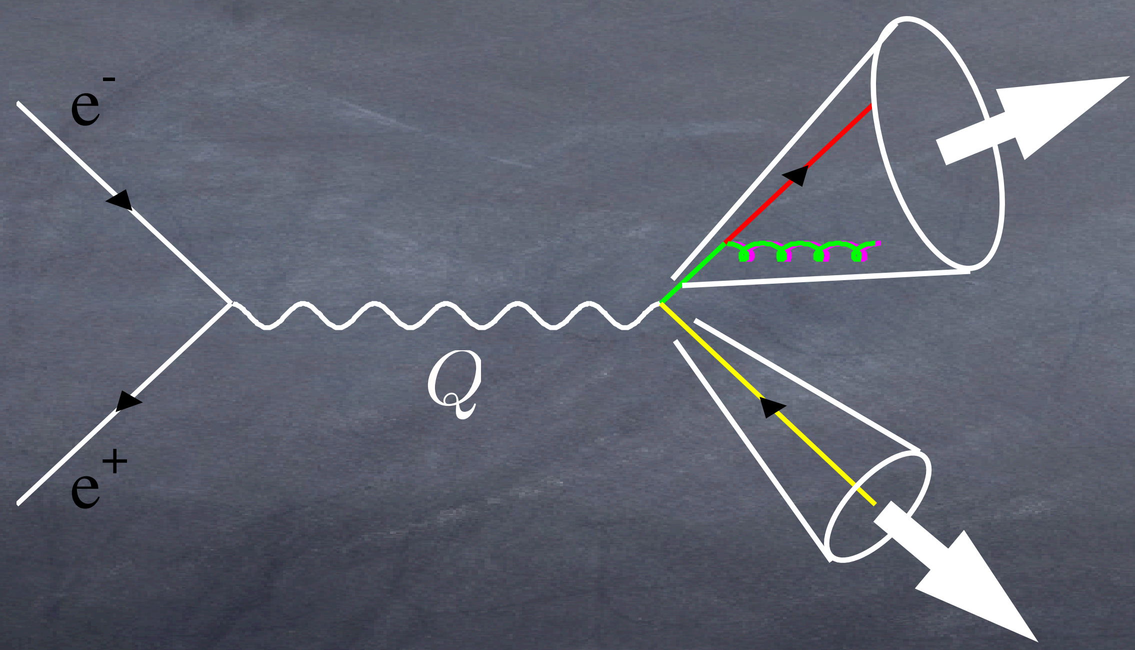


why does a collinear framework do so well?

nature prefers collinear emission?

2.1 FFs and jets

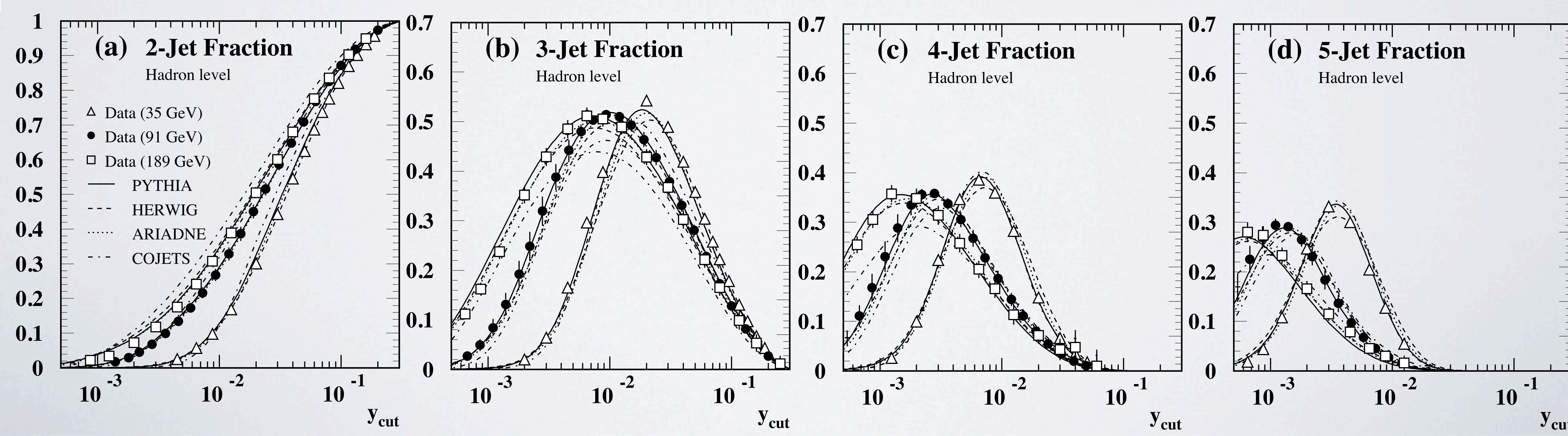
A nice example: $\hat{\sigma}(e^+e^- \to q\bar{q}g)$



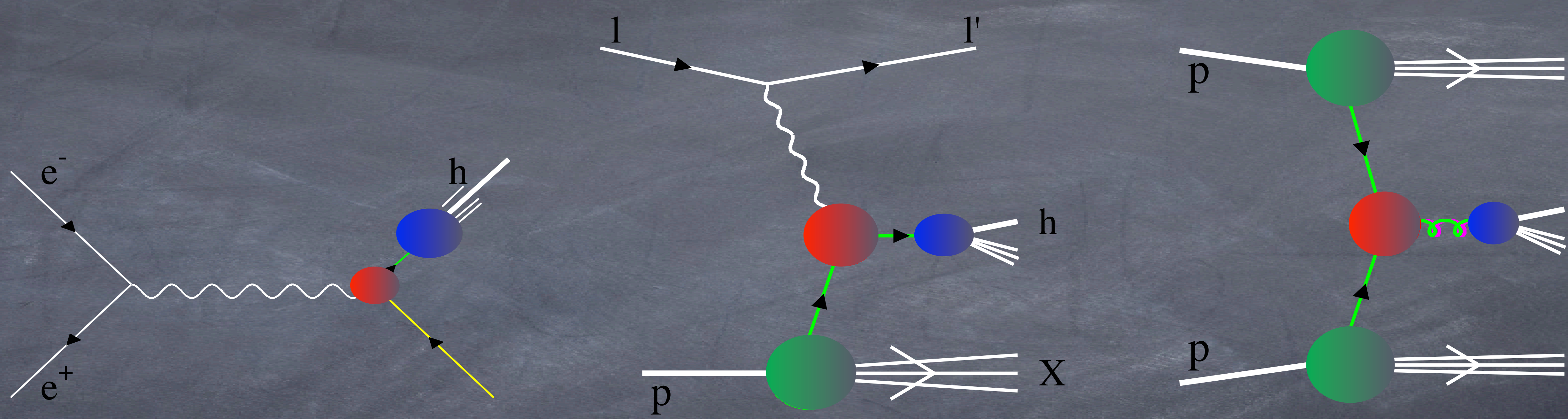
$$z_q \equiv \frac{2E_q}{Q} \qquad z_{\bar{q}} \equiv \frac{2E_{\bar{q}}}{Q} \qquad z_g \equiv \frac{2E_g}{Q}$$

$$z_q + z_{\bar{q}} + z_g = 2$$

$$\frac{d\hat{\sigma}}{dz_q dz_{\bar{q}}} = \sigma_0 \frac{2\alpha_s}{3\pi} \frac{z_q^2 + z_{\bar{q}}^2}{(1-z_q)(1-z_{\bar{q}})}$$



2.2 SIA, SIDIS and pp



$d\sigma = \hat{\sigma}_i \otimes D_i^h$ $d\sigma = f_i \otimes \hat{\sigma}_{ij} \otimes D_j^h$ $d\sigma = f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_k^h$

$$e^+e^- \to (\gamma,\, Z) \to H$$

clean: only FFs

very precise LEP, Belle, BaBar data

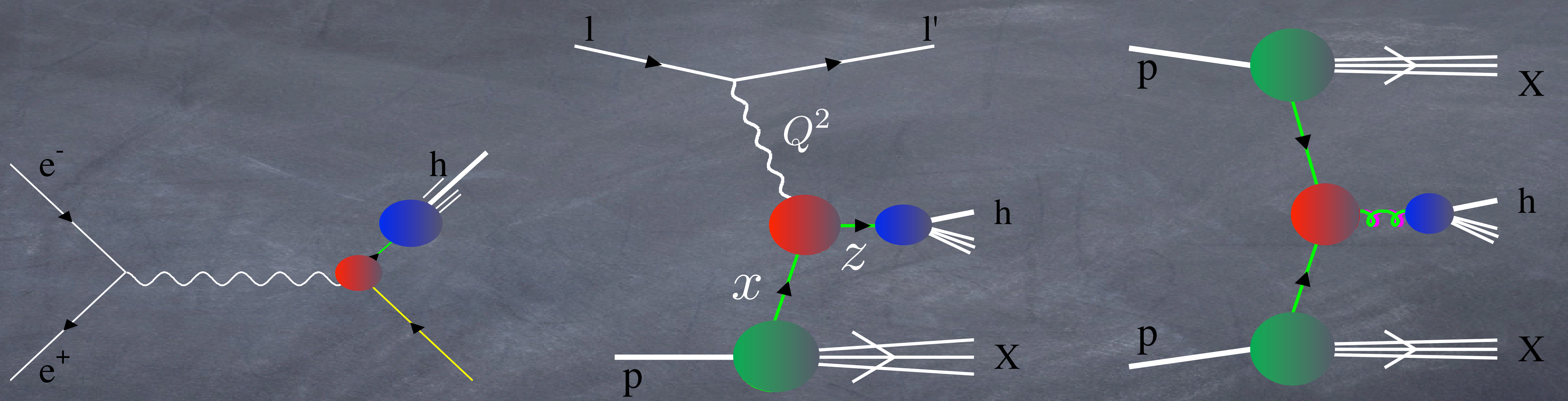
tagged heavy flavors

only information on $D_q^H(z,Q^2) + D_{\bar{q}}^H(z,Q^2)$

"singlet" $\Sigma \equiv D_u^H + D_{\bar{u}}^H + D_d^H + D_{\bar{d}}^H + D_s^H + D_{\bar{s}}^H + \ldots$

suppressed gluon $\frac{\alpha_s(Q^2)}{2\pi} D_g^H(z,Q^2)$

2.2 SIA, SIDIS and pp



$$d\sigma = \hat{\sigma}_i \otimes D_i^h \qquad d\sigma = f_i \otimes \hat{\sigma}_{ij} \otimes D_j^h \qquad d\sigma = f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_k^h$$

$$l + p \longrightarrow l' + H + X$$

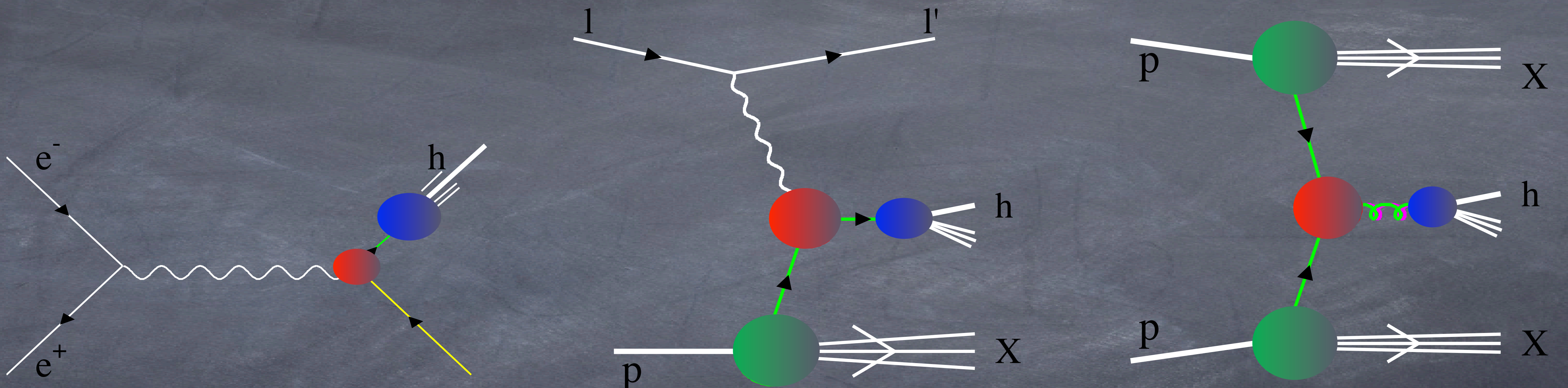
$$F_1^H(x,z,Q^2) = \frac{1}{2}\sum_{q,\bar{q}} e_q^2 f_q(x,Q^2) D_q(z,Q^2)$$

NLO

effective charge: separation

$$+\frac{\alpha_s}{2\pi}\sum_{q,\bar{q}} e_q^2 [f_q \otimes C_{qq} \otimes D_q^H + f_q \otimes C_{gq} \otimes D_g^H + f_g \otimes C_{qg} \otimes D_q^H]$$

2.2 SIA, SIDIS and pp



$$d\sigma = \hat{\sigma}_i \otimes D_i^h \qquad d\sigma = f_i \otimes \hat{\sigma}_{ij} \otimes D_j^h \qquad d\sigma = f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_k^h$$

$$l + p \longrightarrow l' + H + X$$

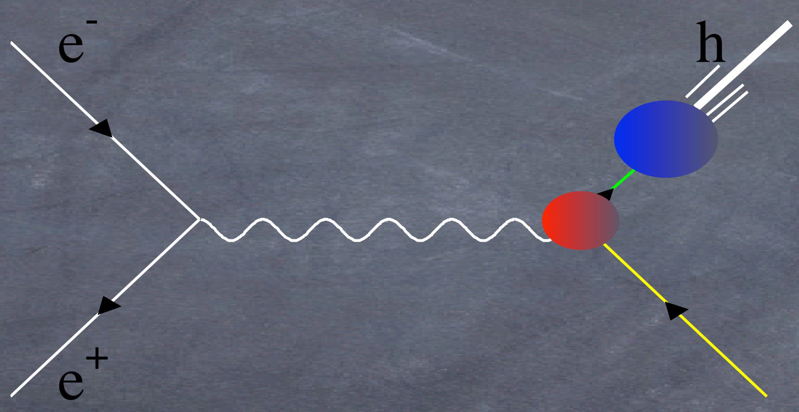
charge and flavor separation

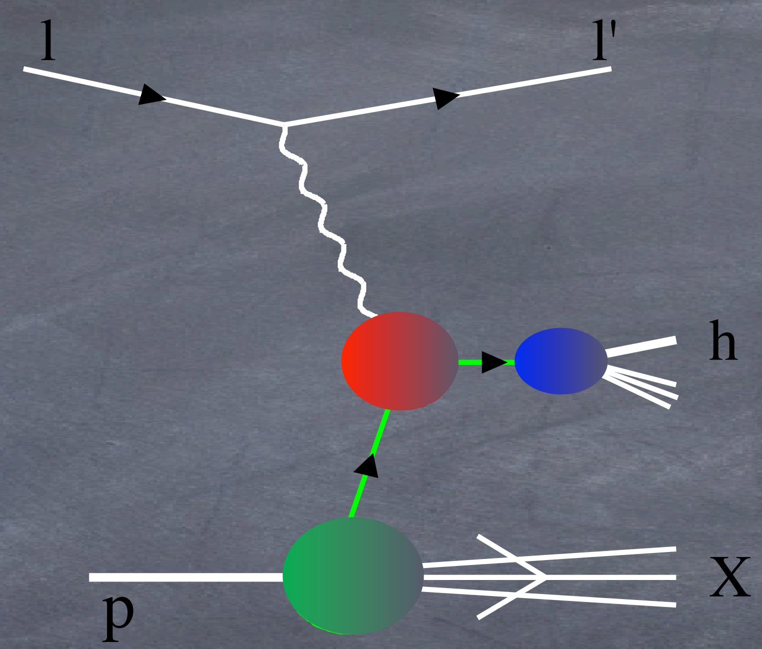
complementary z,Q^2 ranges

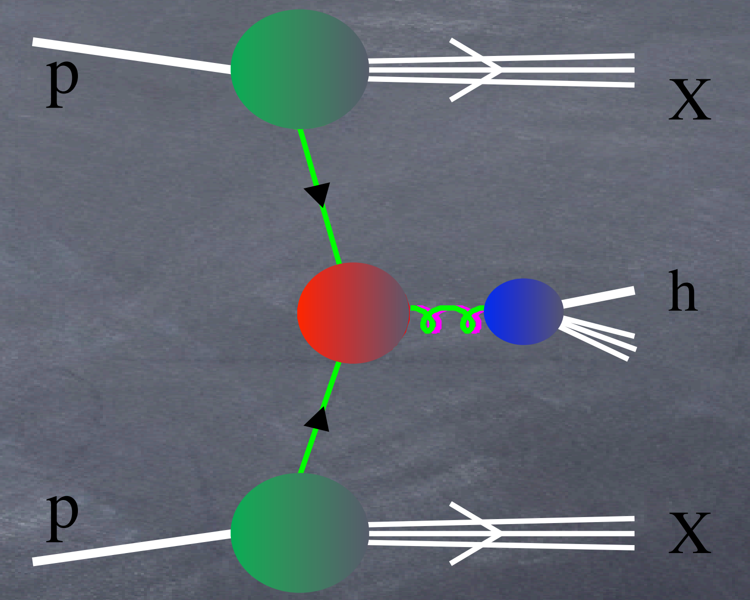
mainly light flavors

requires precise PDFs!

2.2 SIA, SIDIS and pp





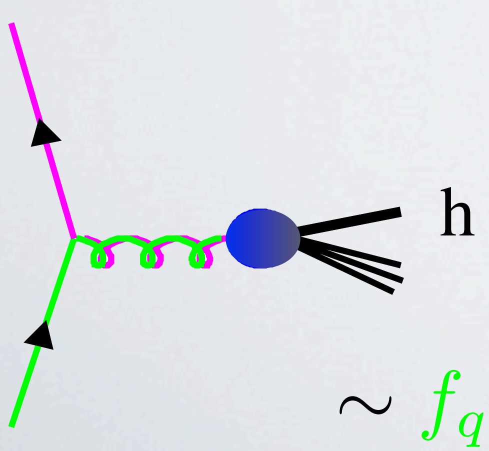


$$d\sigma = \hat{\sigma}_i \otimes D_i^h$$

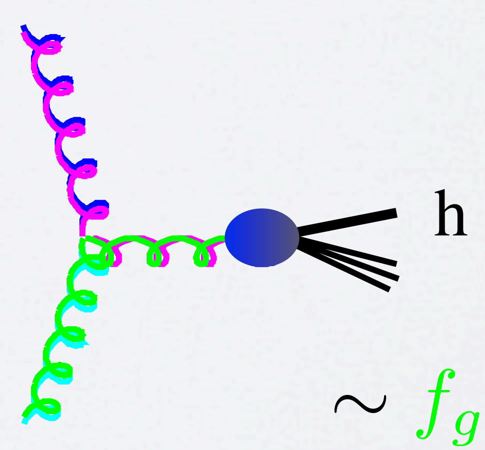
$$d\sigma = f_i \otimes \hat{\sigma}_{ij} \otimes D_j^h$$

$$d\sigma = f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_k^h$$

$$p + p \longrightarrow H + X$$

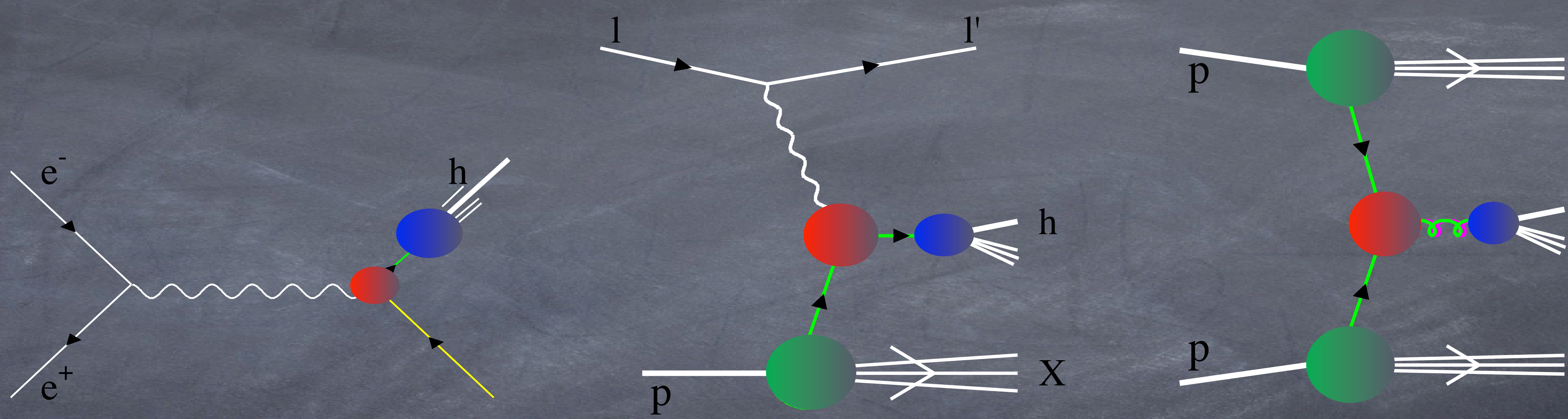


$$\sim f_q \otimes f_{\overline{q}} \otimes C_{q\overline{q}g} \otimes D_g^h$$



$$\sim f_g \otimes f_g \otimes C_{ggg} \otimes D_g^h$$

2.2 SIA, SIDIS and pp



$d\sigma = \hat{\sigma}_i \otimes D_i^h$

$d\sigma = f_i \otimes \hat{\sigma}_{ij} \otimes D_j^h$

$d\sigma = f_i \otimes f_j \otimes \hat{\sigma}_{ijk} \otimes D_k^h$

$p + p \longrightarrow H + X$

large and direct gluon contribution

several sub-processes

RHIC Tevatron LHC benchmarks

requires precise PDFs

"convoluted"

large TH uncertainty

2.2 SIA, SIDIS and pp

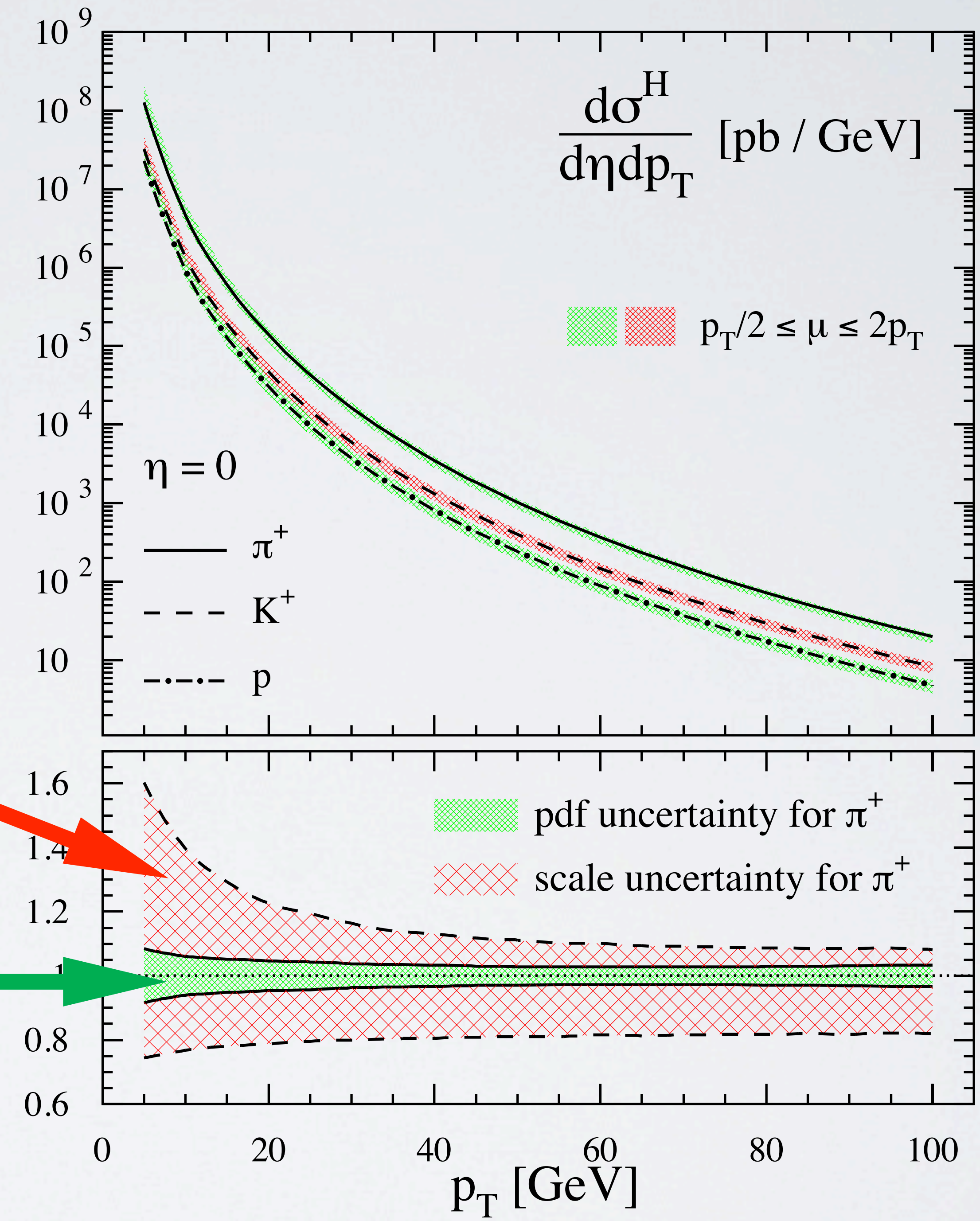
example: pp@7 TeV

pQCD uncertainties

> 50% pT< 5GeV

PDFs uncertainties

5-10%



nevertheless our best grip on gluons...

2.2 SIA, SIDIS and pp

example: pp@7 TeV

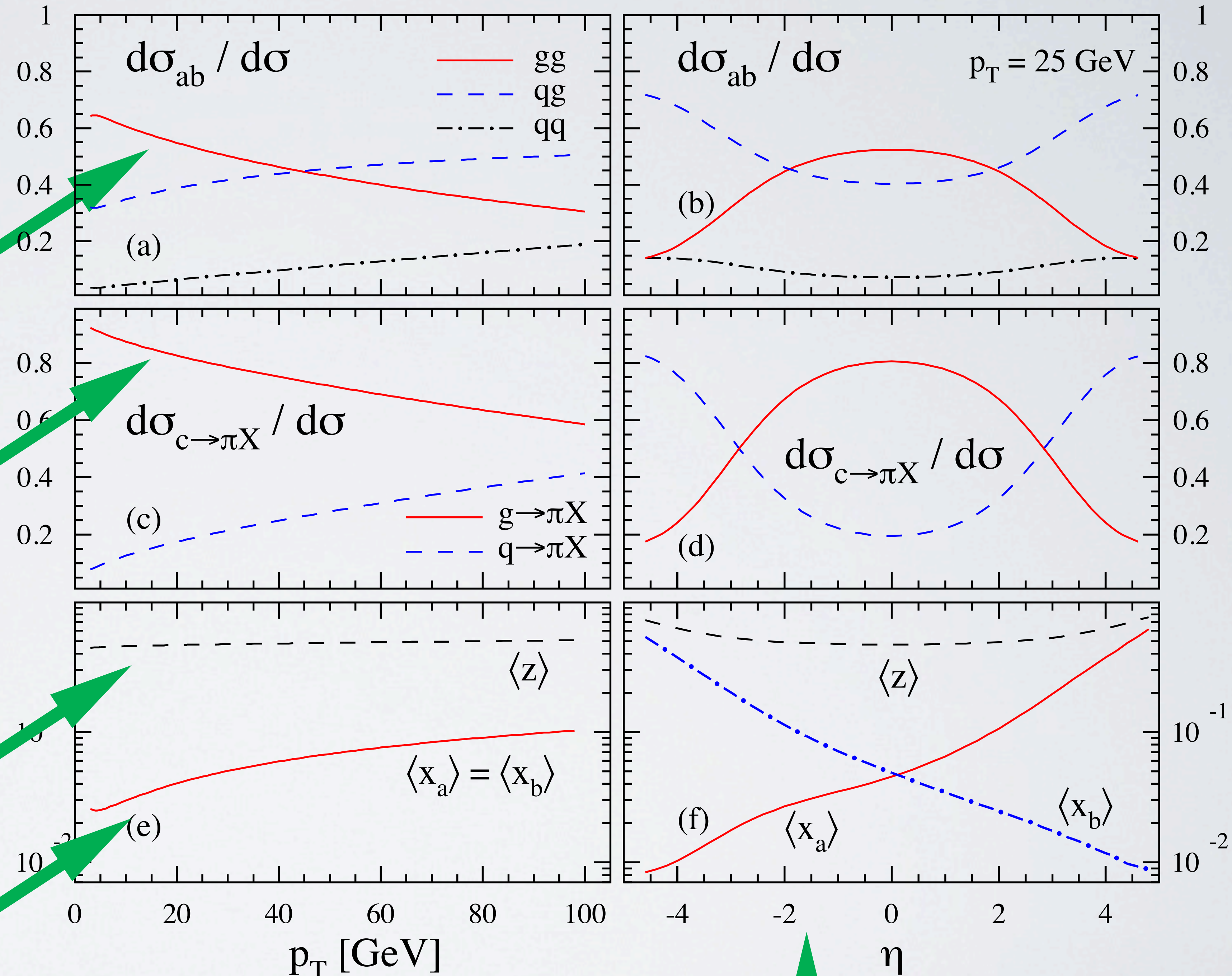
"convoluted"

gluon PDF at low pT
but....

gluon FF everywhere

range in z (<z>~0.45)

range in x (<x>~0.01-0.1)



at fixed pT η selects

2.2 SIA, SIDIS and pp

$$\frac{1}{2\pi i}\int_{\mathcal{C}_n} dn\, z_c^{-n}\, D_c^n$$

multiple convolutions

time consuming

$$d\sigma = \sum_{abc} \int\int\int f_a f_b\, d\hat{\sigma}_{ab\to cX}\, D_c\, dx_a dx_b dz_c$$

Mellin space: $D_i^n(\mu^2) = \int_0^1 dz\, z^{n-1}\, D_i(z,\mu^2)$

~ O(10[2]) moments accurate representation of FFs (and PDFs)

transforms: convolutions into products (of numbers)

differential equations into linear equations

fast and easy
if known functional form
(known analytic continuation)

back to z space (inversion) very fast (complex analysis)

$$d\sigma = \frac{1}{2\pi i}\sum_{abc} \underbrace{\int_{\mathcal{C}_n} dn}_{\text{Mellin inversion}}\; \underbrace{D_c^n}_{\text{FIT}}\; \underbrace{\int\int\int z_c^{-n} f_a f_b\, d\hat{\sigma}_{ab\to cX}\, dx_a dx_b dz_c}_{\text{Precompute once and save grids}}$$

2.3 Global FFs fits: DSS

several FFs parameterizations for π, K, p *and* $h^{\pm}$ $(\Lambda, \eta, D_c ...)$

CGGRW(1994), BFGW(2000) — Bourhis et al (2001)

BKK(1995), KKP(2000), AKK(2005,2008) *include pp* — Albino, Kniehl, Kramer (1995-2008)

KRE(2000) — Kretzer (2000)

HKNS(2007) — Hirai, Kumano, Nagai, Sudoh (2007)

DSS(2007) ← *only global analysis* — de Florian, RS, Stratmann (2007)

LSS(2013) *new SIDIS only* — Leader, Sidorov, Stamenov (2013)

SIA-only fits: no charge separation or ad-hoc assumption

$$D_{\bar{q}}^{h^+}(z, Q^2) = (1 - z)\, D_q^{h^+}(z, Q^2)$$

2.3 Global FFs fits: DSS

fragmentation functions for $\pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \overline{K}^0, p, \overline{p}, n, \overline{n}, h^+, h^-$

$h^+ = \pi^+ + K^+ + p + res^+$

LO and NLO global fits available

SIA data includes: TPC, TASSO, SLD, ALEPH, DELPHI, OPAL + "flavor" tag

SIDIS data from : HERMES, EMC(h) $Q > 1\,\mathrm{GeV}$

pp data from : PHENIX, STAR, BRAHMS, CDF, UA1, UA2 $p_T > 1\mathrm{GeV}$ (η)

Estimation of uncertainties using Lagrange multipliers (2007)

Improved Hessian approach (2012)

Data and error estimate update (2014) DSS II

2.3 Global FFs fits: DSS

- Flexible parametrization

$$D_i^H(z, Q_0^2) = N_i \, z^{\alpha_i} (1-z)^{\beta_i} \left[1 + \gamma_i (1-z)^{\delta_i}\right]$$

at initial scale

$$\begin{aligned} Q_0^2 &= 1\,\mathrm{GeV}^2 \qquad u, d, s, g \\ Q_0^2 &= m_Q^2 \qquad c, b \end{aligned}$$

with

α_s and Λ_{QCD} from MRST 2002

- Try to avoid Isospin symmetry assumptions
  - *allow for possible breaking of SU(3) of sea and SU(2) in favored distributions*
  - *unless data can not discriminate for unfavored fragmentations*

$$D_{d+\bar{d}}^{\pi^+} = N \, D_{u+\bar{u}}^{\pi^+}$$

$$D_s^{\pi^+} = D_{\bar{s}}^{\pi^+} = N' D_{\bar{u}}^{\pi^+}$$

$$D_{\bar{u}}^{\pi^+} = D_d^{\pi^+}$$

$$D_{\bar{u}}^{K^+} = D_s^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^+}$$

- Normalizations for different experiments (if not included in syst.)

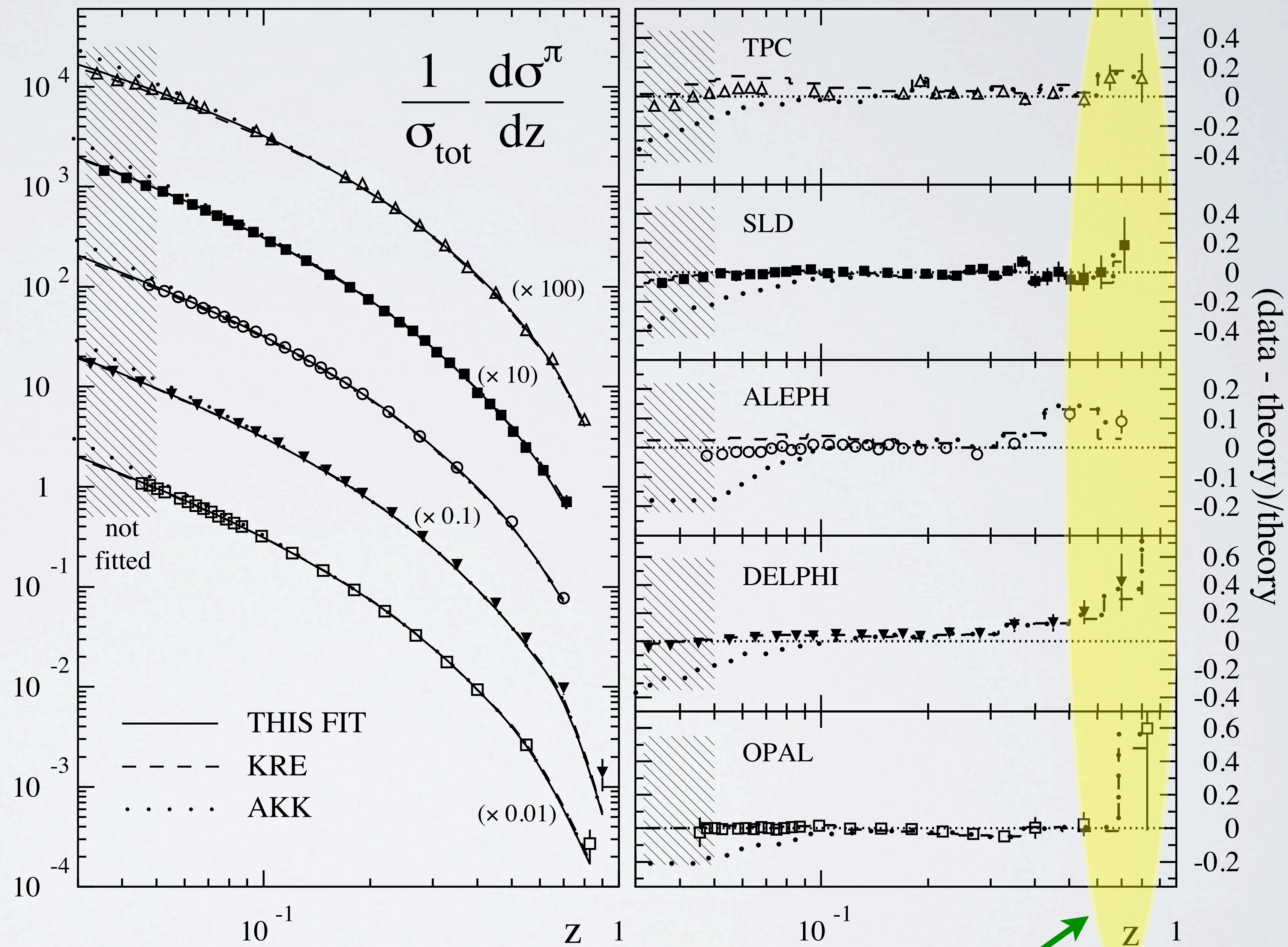
2.3 Global FFs fits: DSS

SIA: still works very well within the global fit

factorization ✔

universality ✔

z-cut z>0.05
(0.1 for Kaons/Protons)



large errors at z > 0.5

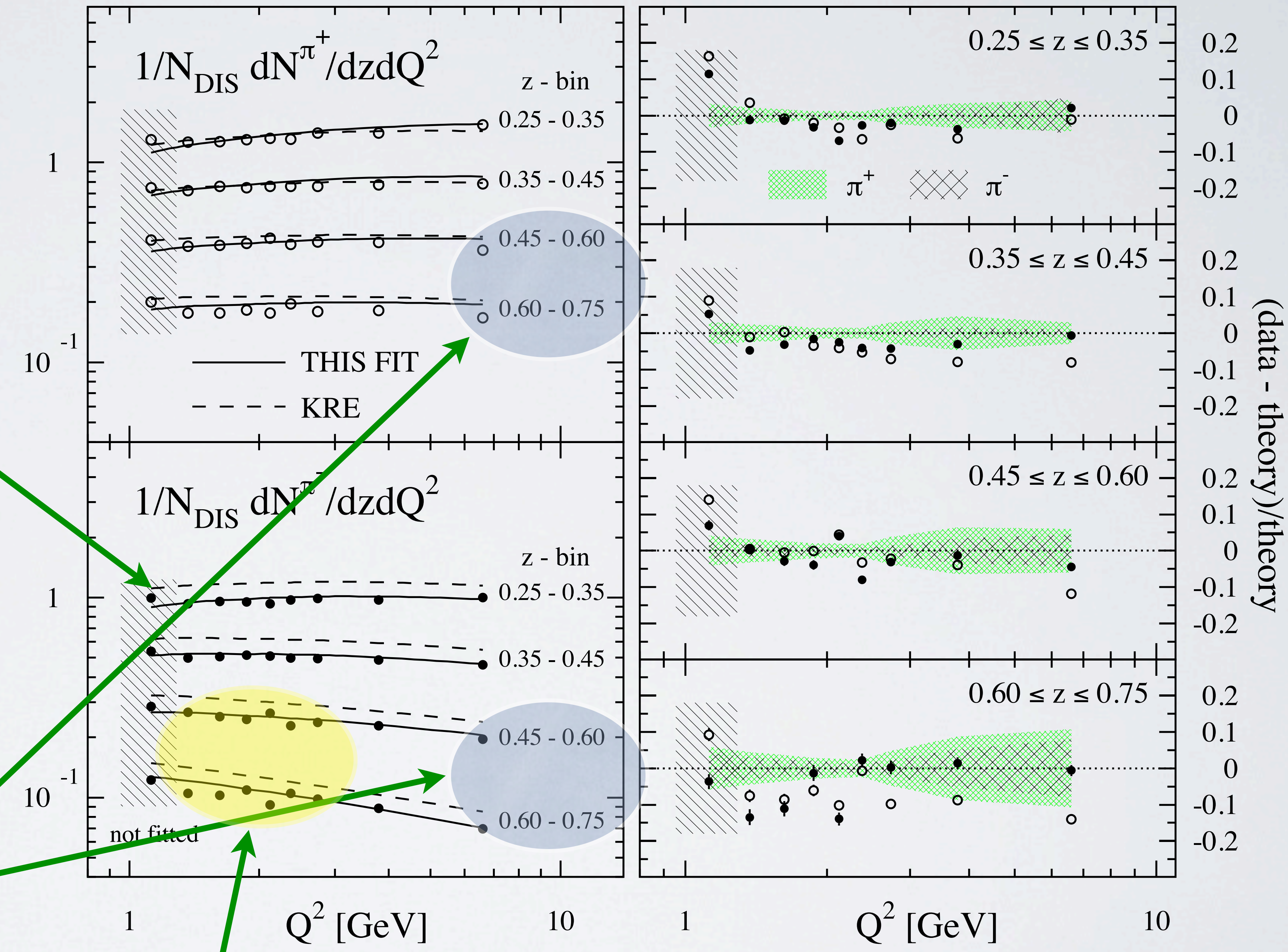
2.3 Global FFs fits: DSS

Good description of SIDIS multiplicities

ad-hoc charge separation from Kretzer fails

large z covered

Hermes data (not final)



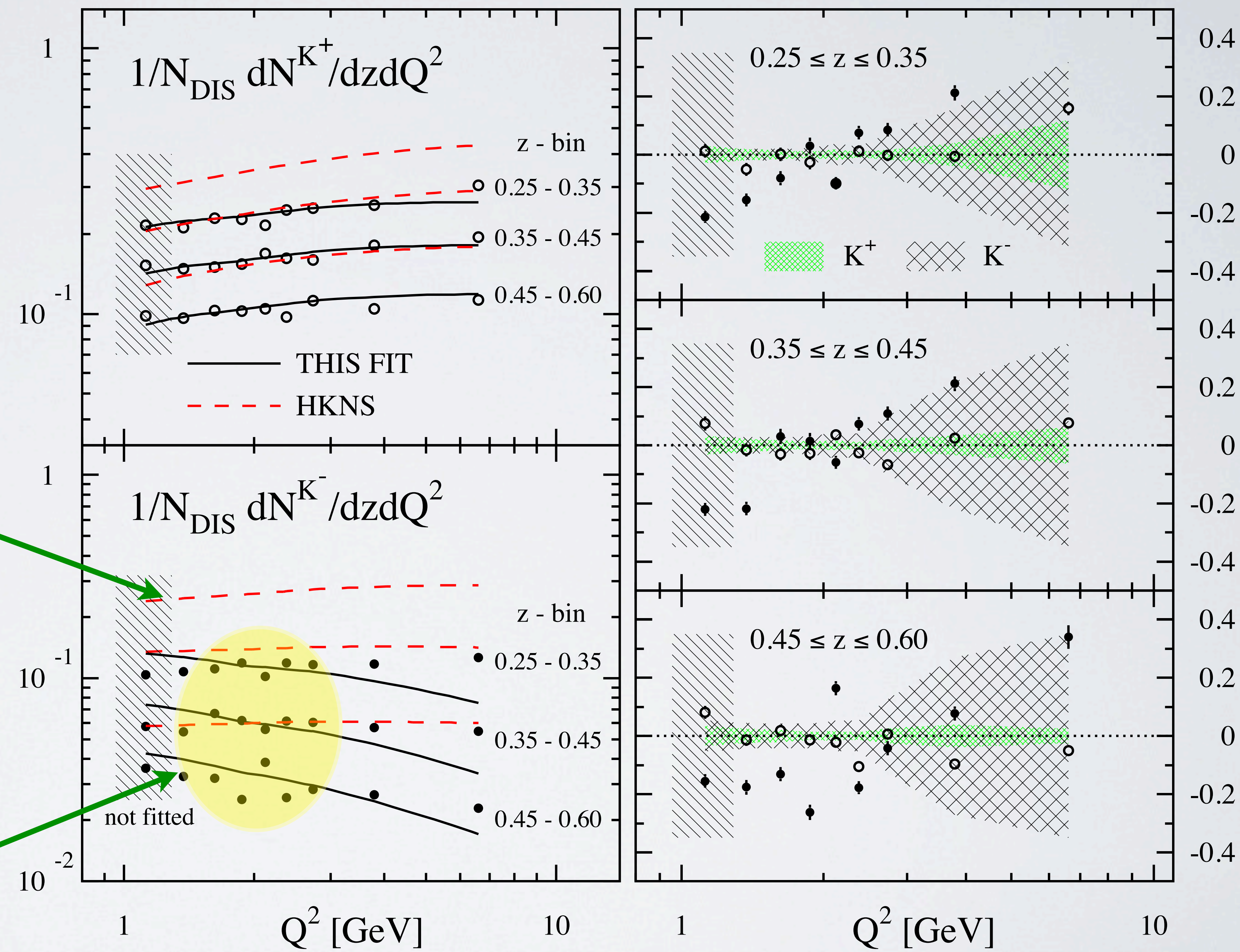
hard to fit!

2.3 Global FFs fits: DSS

Not so nice for Kaons

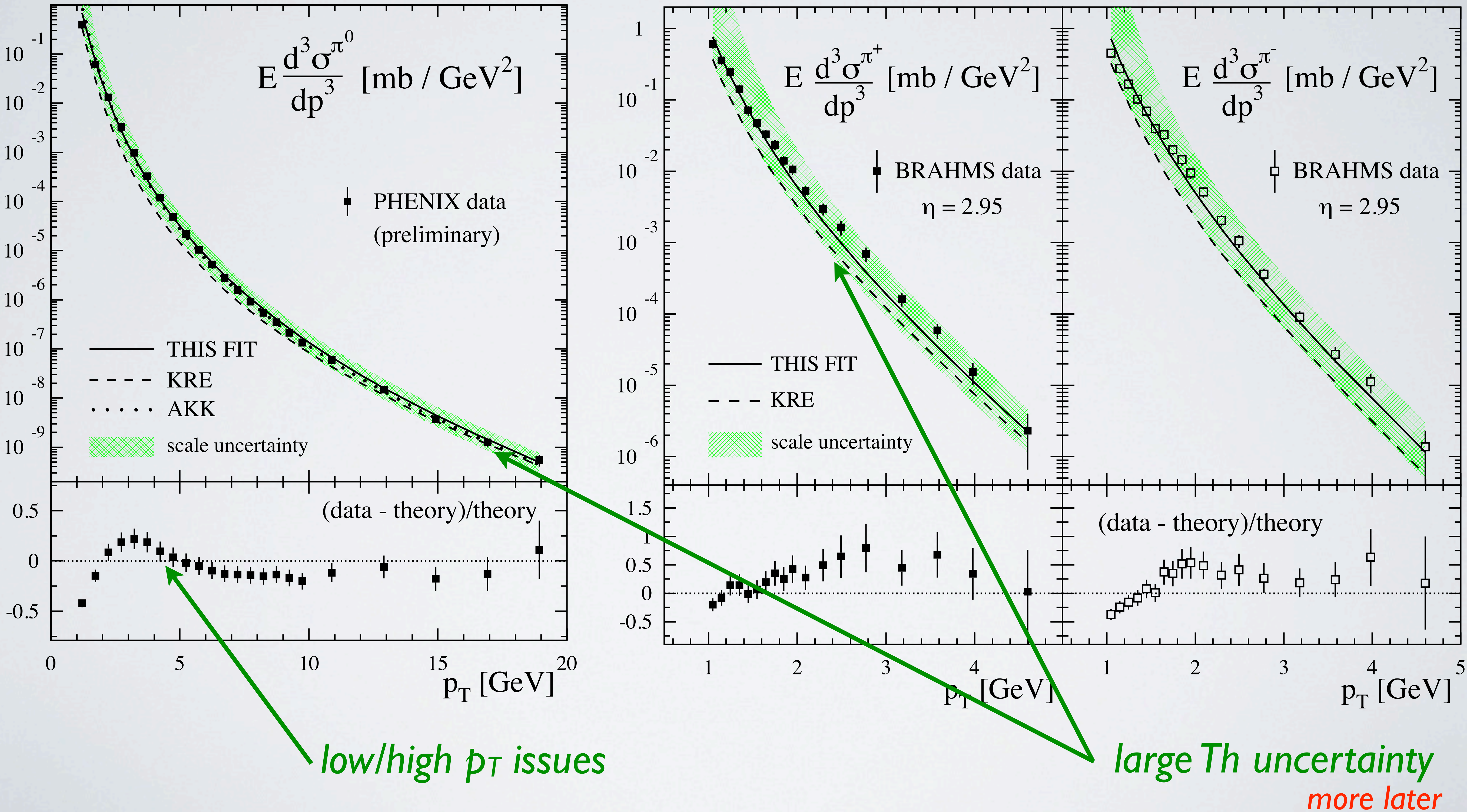
ad-hoc charge separation
from HKNS fails
factor of 2 or 3!!

even harder to fit!



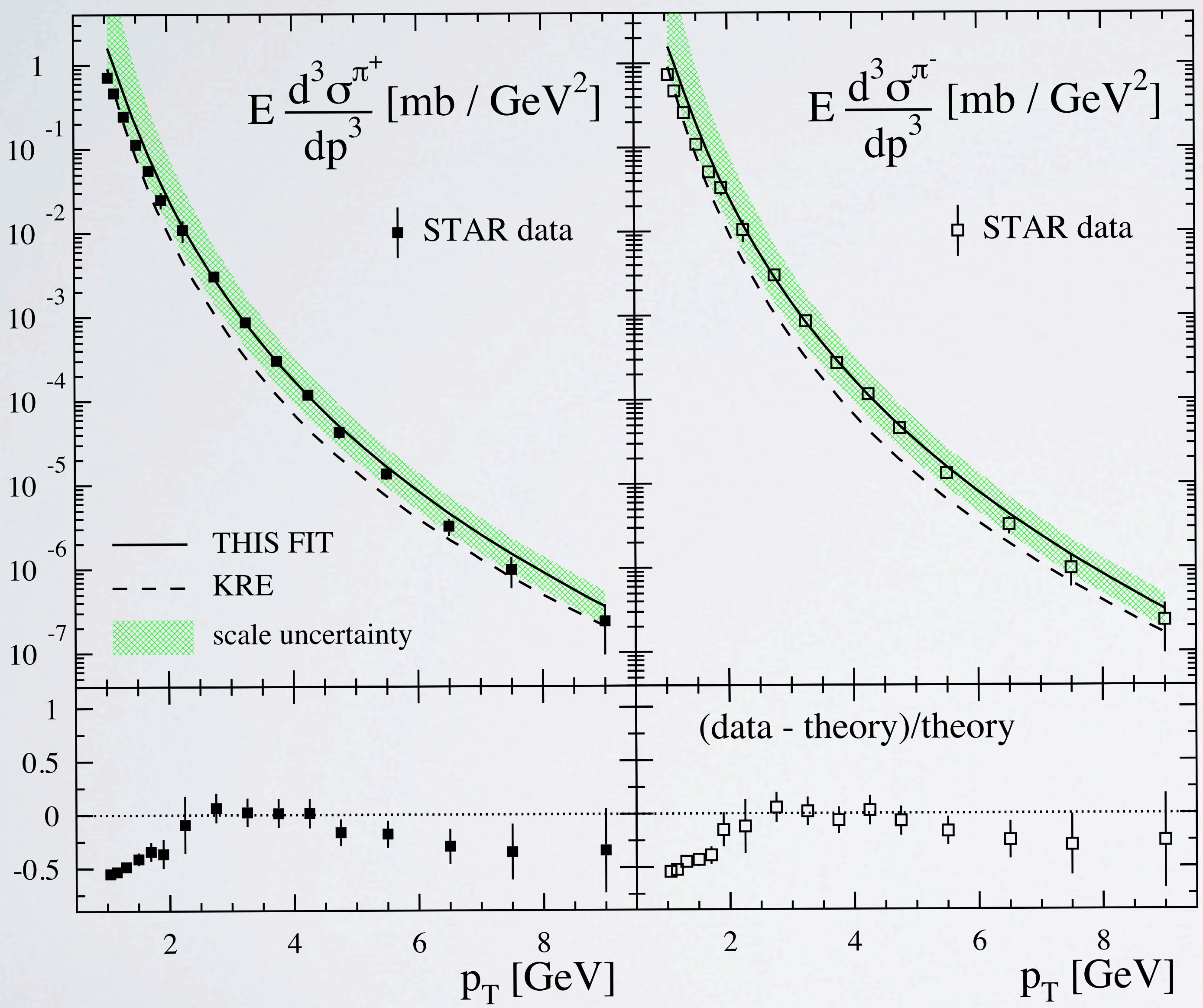
2.3 Global FFs fits: DSS

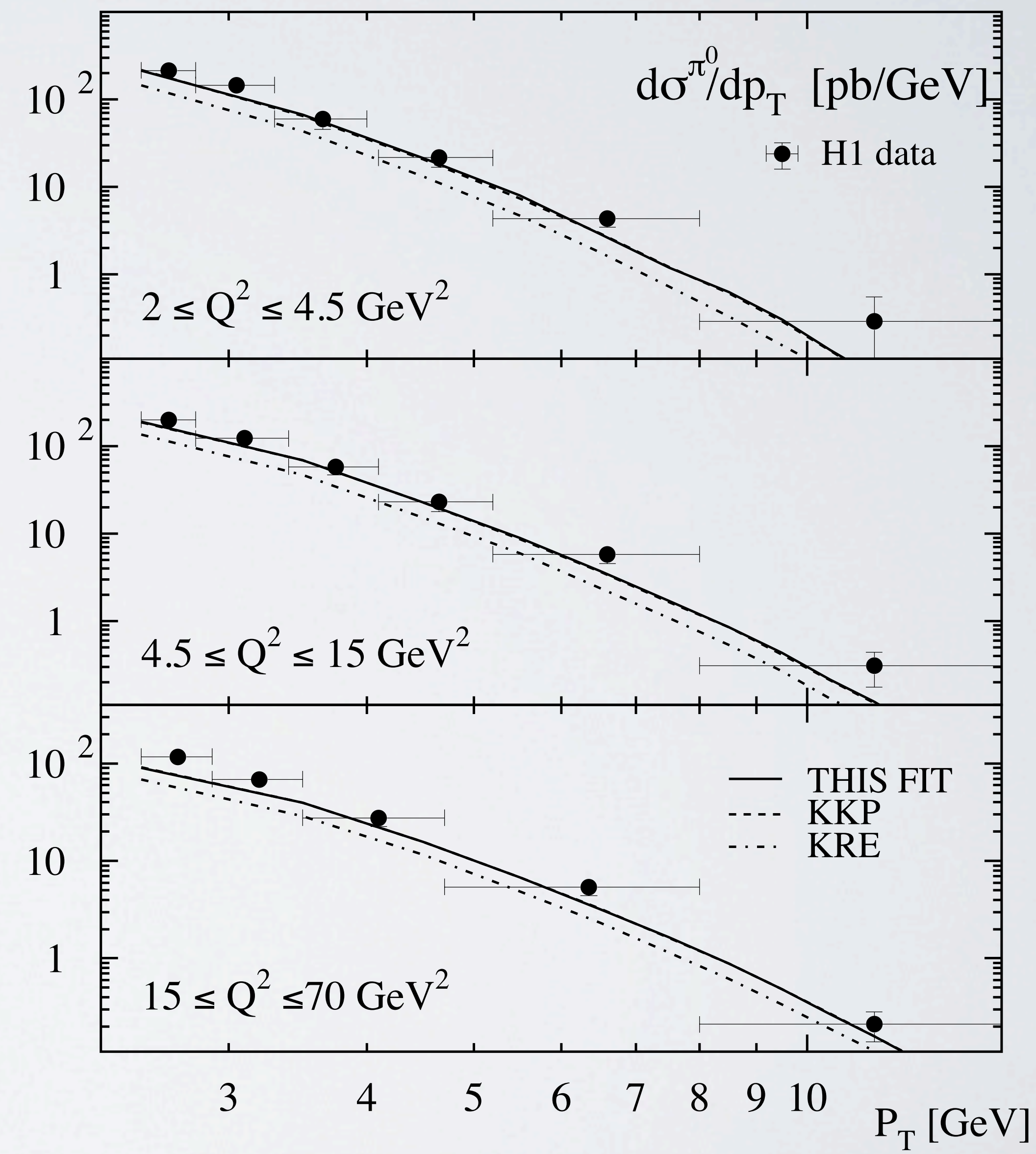
pp data: look impressive (in log scale)



2.3 Global FFs fits: DSS

Agreement with data not included in the fit





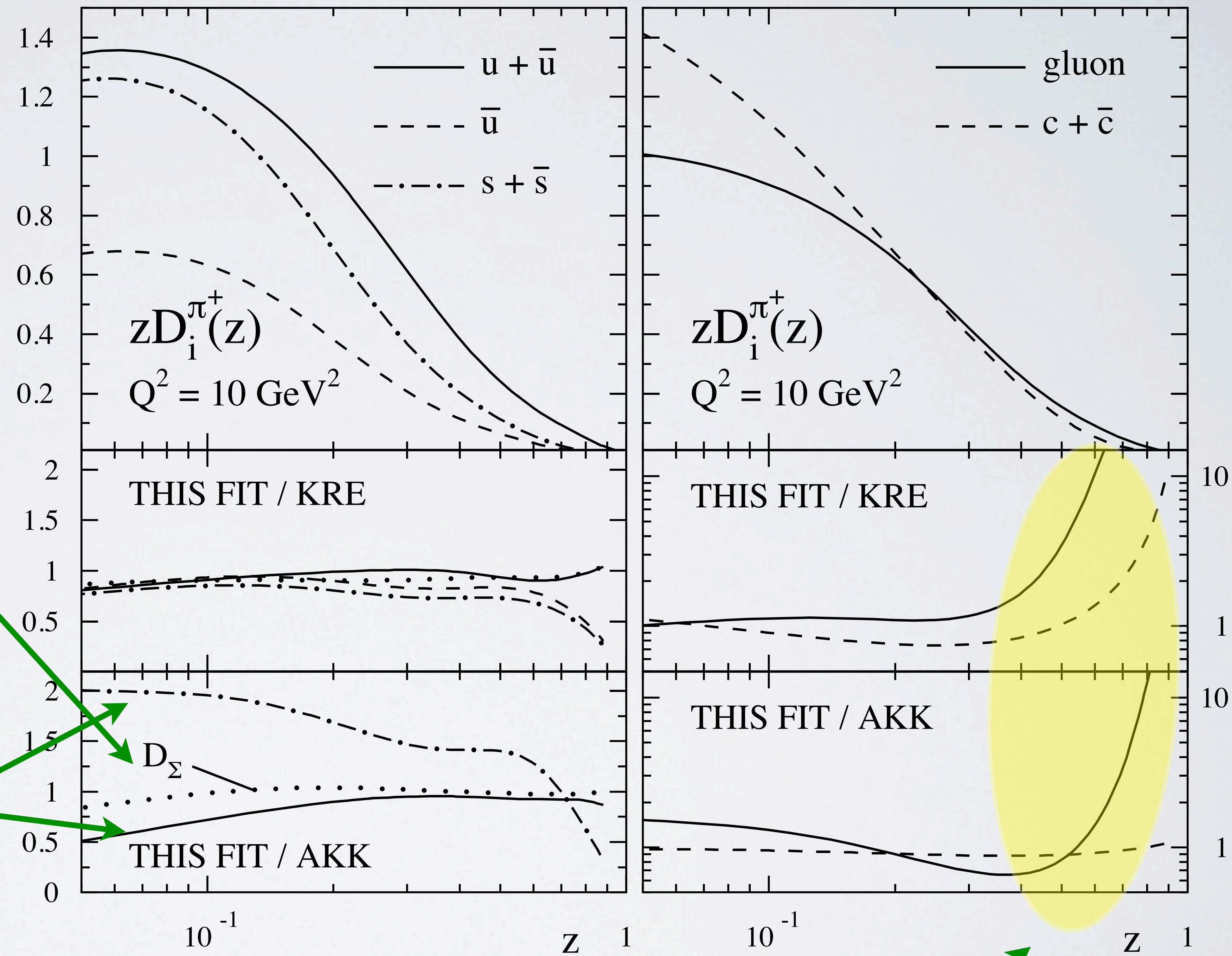
STAR charged pions: forgotten!

pT-dependent SIDIS: too difficult!

2.3 Global FFs fits: DSS

similar singlet: SIA

differences in sea: explains SIDIS and pp



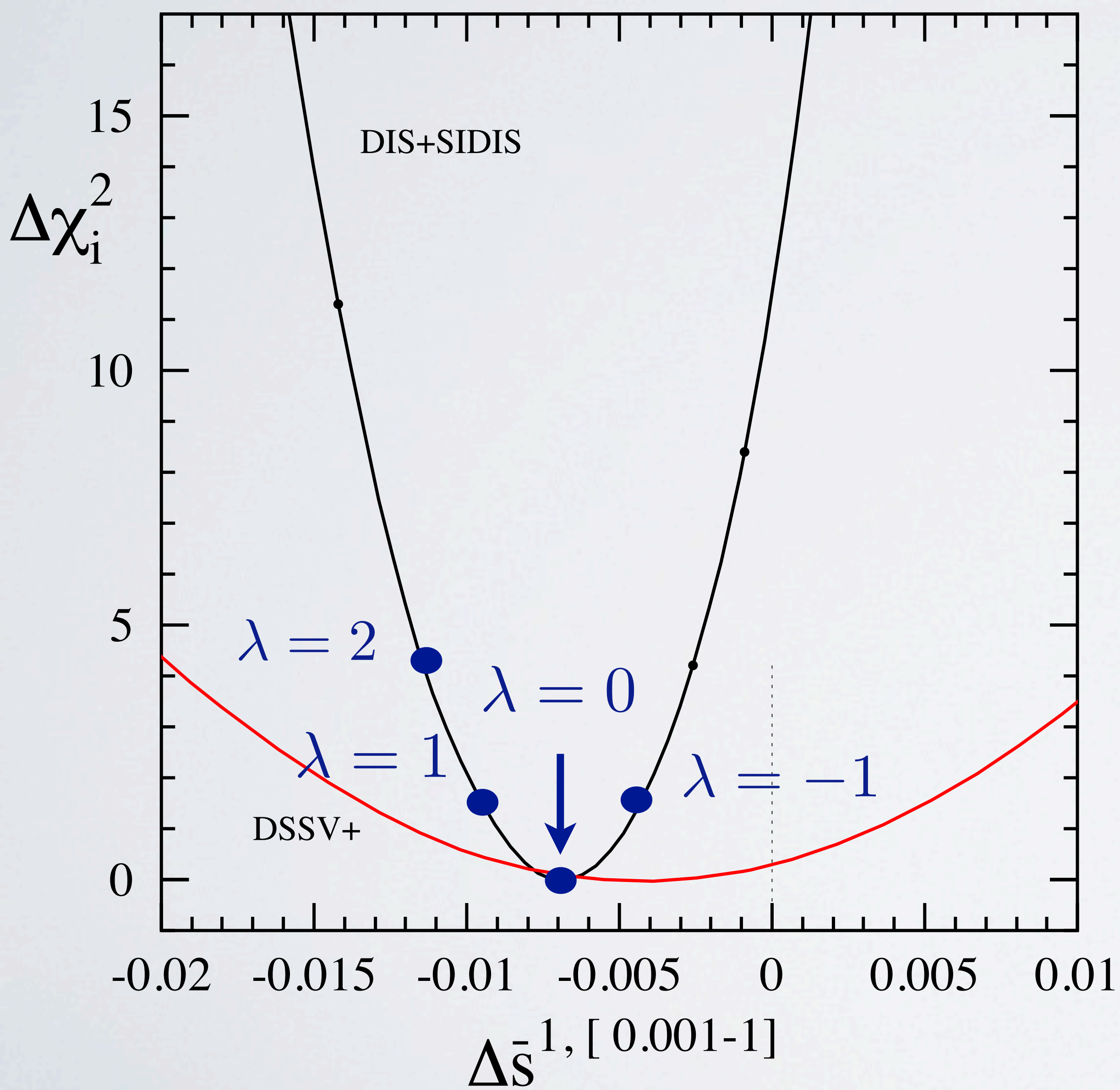
huge differences in the gluon at large z : pp

2.4 Uncertainties: Lagrange Multipliers

DSS use LM technique to estimate uncertainties (from exp. errors)

most robust method: no assumptions on $\chi^2(\{a_i\})$ or $\mathcal{O}(\{a_i\})$

just compute it!



minimize:

$$\Phi(\lambda_i, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i \, \mathcal{O}_i(\{a_j\})$$

See how fit deteriorates when FFs forced to give different prediction for $\mathcal{O}_i$

$\Delta\chi^2$ *should be parabolic if data set can determine the observable*

what $\Delta\chi^2$?

	68%	90%
parameter fitting	1	2.71
hypothesis testing	$\sim \sqrt{N}$	
"pragmatic"	*whatever it takes to have the fitted data within bands*	

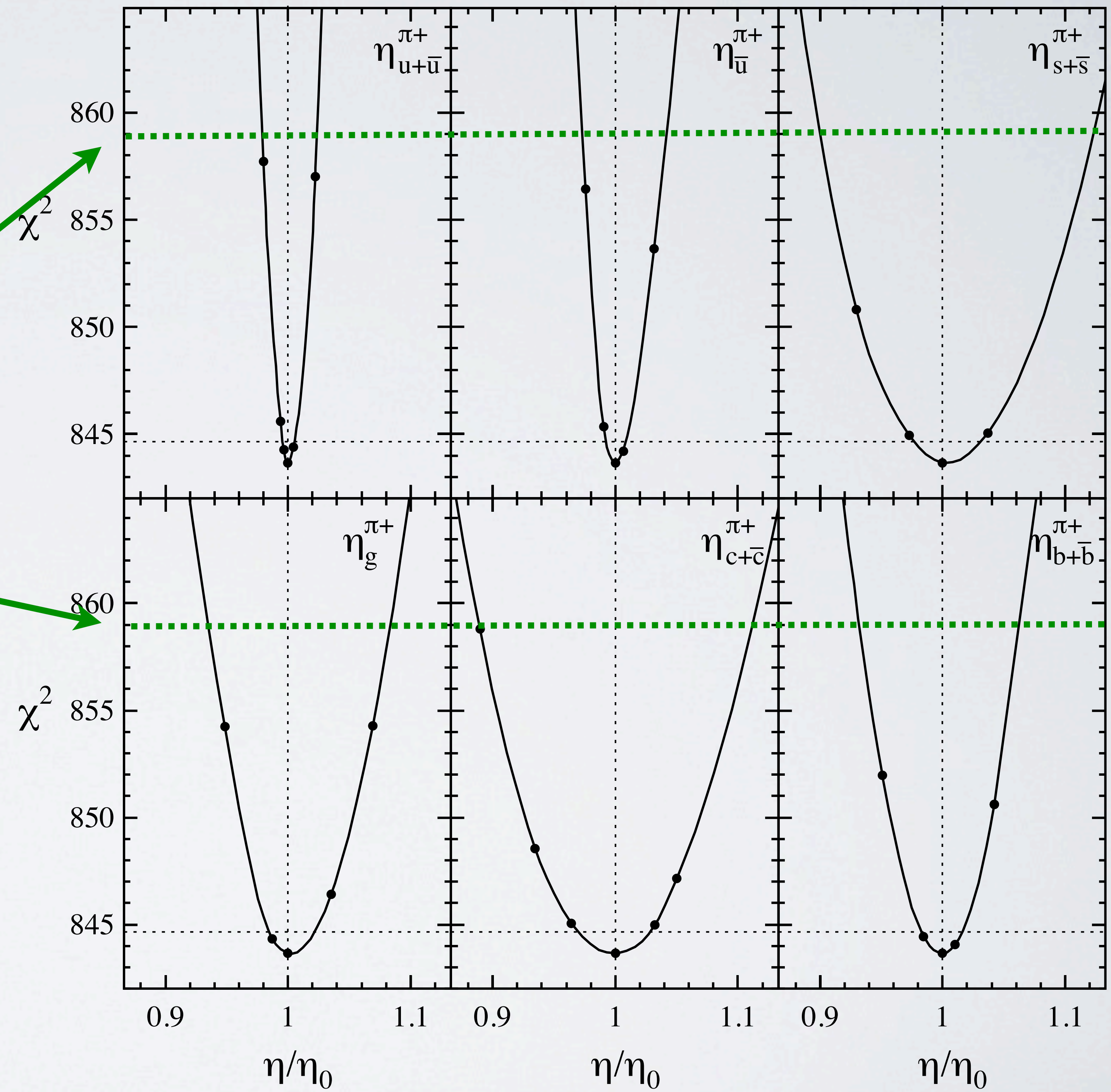
$$\Delta\chi^2_{68} = \xi_{68} - \xi_{50} \qquad \int_0^{\xi_{68}} d\chi^2 \, P_N(\chi^2) = 0.68$$

2.4 Uncertainties: Lagrange Multipliers

example: truncated moments:

$$\int_{0.2}^{1} z\, D_i^H(z, Q = 5\,\mathrm{GeV})\, dz$$

$\Delta\chi^2 = 15(\sim 2\%)$



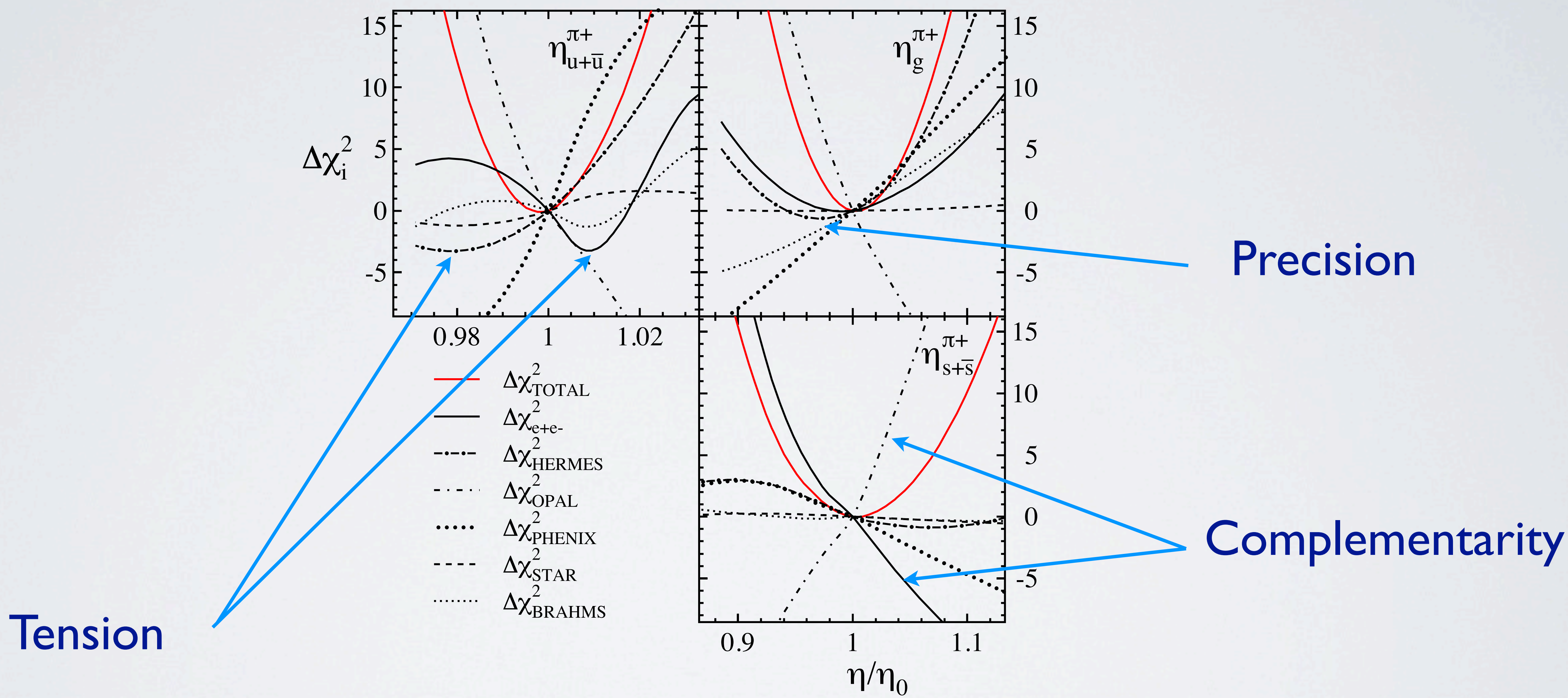
u < 5%   s ~ 10 %

dominated by z~0.2

factor of 2 larger for Kaons   ~ 20-25% for protons

2.4 Uncertainties: Lagrange Multipliers

Individual profiles for data subsets: impact & interplay



Constrained parabola as a result of global fit

not user friendly!

2.5 Uncertainties: Standard Hessian

$$D_q^h(z, p_{T0}^2) \longrightarrow \quad S\{a_i\}$$ set of fit parameters

best fit $\quad S_0\{a_i^0\} \quad \chi_0^2 = \chi^2(\{a_i^0\})$

assuming quadratic profile

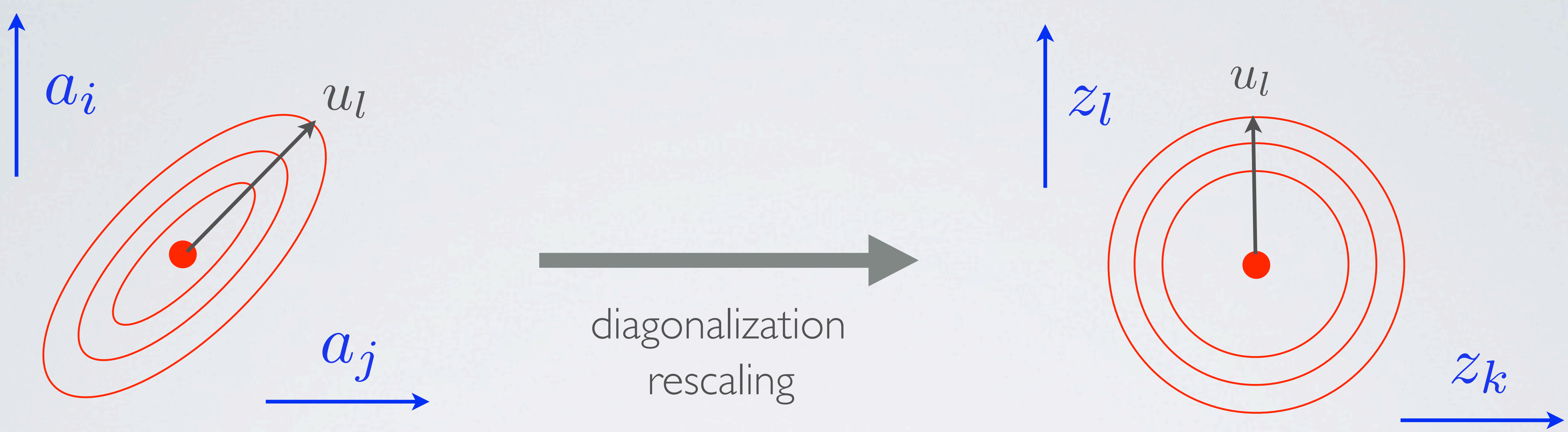
$$\Delta\chi^2 \equiv \chi^2(\{a_i\}) - \chi^2(\{a_i^0\}) \simeq \sum_{ij} H_{ij} y_i y_j$$

$$H_{ij} \equiv \frac{1}{2}\frac{\partial^2\chi^2}{\partial y_i \partial y_j} \qquad y_i \equiv a_i - a_i^0$$

assuming linear propagation

$$\Delta\mathcal{O} \simeq \left[\Delta\chi^2 \sum_{ij} \frac{\partial\mathcal{O}}{\partial y_i}(H^{-1})_{ij}\frac{\partial\mathcal{O}}{\partial y_j}\right]^{1/2}$$

2.5 Uncertainties: Improved Hessian



$$\sum_j H_{ij}\, v_{jk} = \epsilon_k v_{ik} \qquad a_i - a_i^0 = \sum_k v_{ik}\, s_k\, z_k \qquad \Delta\chi^2 = \sum_k z_k^2$$

$S_k^{\pm}$ *Hessian eigenvector sets*

$$\Delta\mathcal{O} \simeq \left[\frac{1}{2}\sum_k \left[\mathcal{O}(S_k^+) - \mathcal{O}(S_k^-)\right]^2\right]^{1/2}$$

help with correlations/variations
easier/faster than LM
replicas for re-weighting

2.6 DSS-II

DSS + M. Epele, R. Hernández Sanchez 14xx.xxxx

mostly a DSS fit but with new SIA, SIDIS, RHIC and LHC data (pions)

analytic normalization

$$\chi^2 = \sum_j \sum_i \left(\frac{\sigma_i^{exp} - \mathcal{N}_j \sigma_i^{th}}{\Delta\sigma_i^{exp}} \right)^2 + \sum_j \left(\frac{1 - \mathcal{N}_j}{\Delta\sigma_j^{exp}} \right)^2$$

flexible parameterizations

$$D_i^{\pi^+}(z, Q_0^2) = N_i z^{\alpha_i} \left[(1-z)^{\beta_i} + \gamma_i (1-z)^{\delta_i} \right]$$

$$D_{\bar{s}}^{\pi^+}(z, Q_0^2) = N_s z^{\alpha_s} D_{\bar{u}}^{\pi^+}(z, Q_0^2)$$

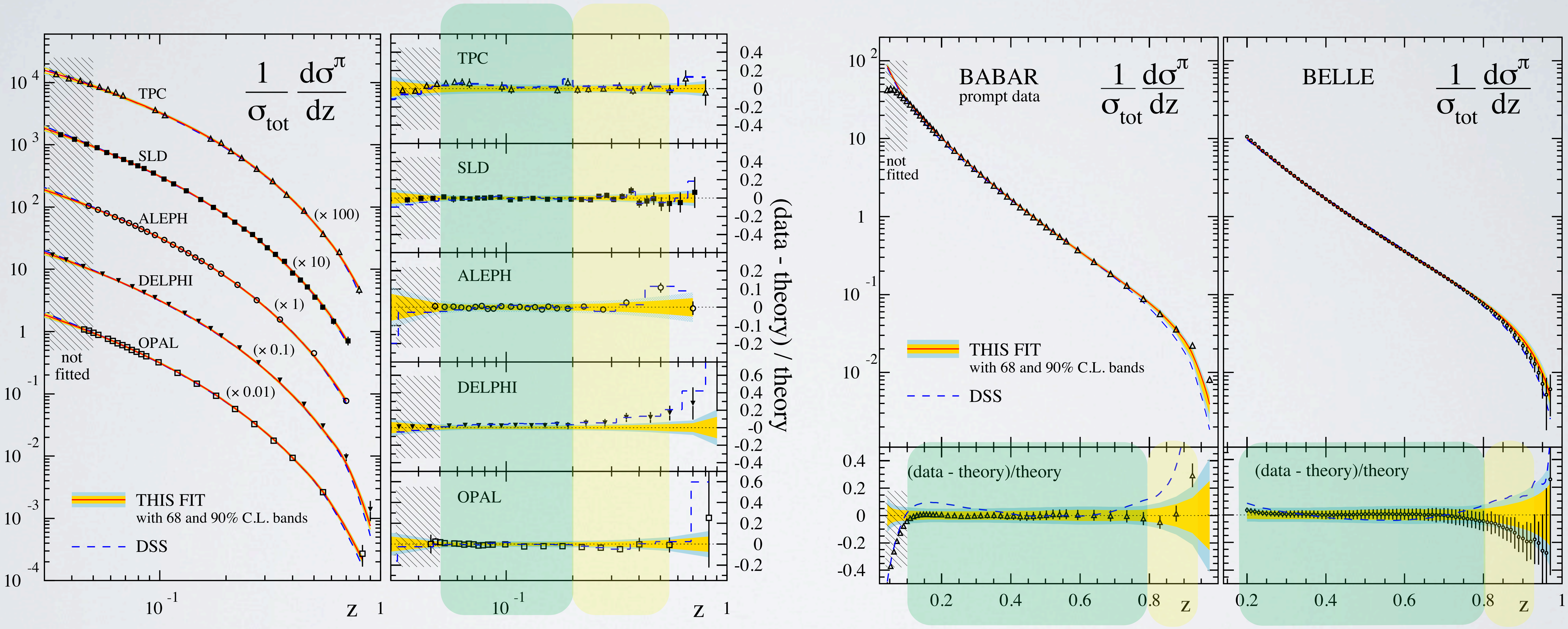
MSTW input PDFs and alpha_s

pT cuts in pp data

68 and 90 % CL error estimates (improved hessian)

New SIA

new Belle and BaBar (large z, lower Q)

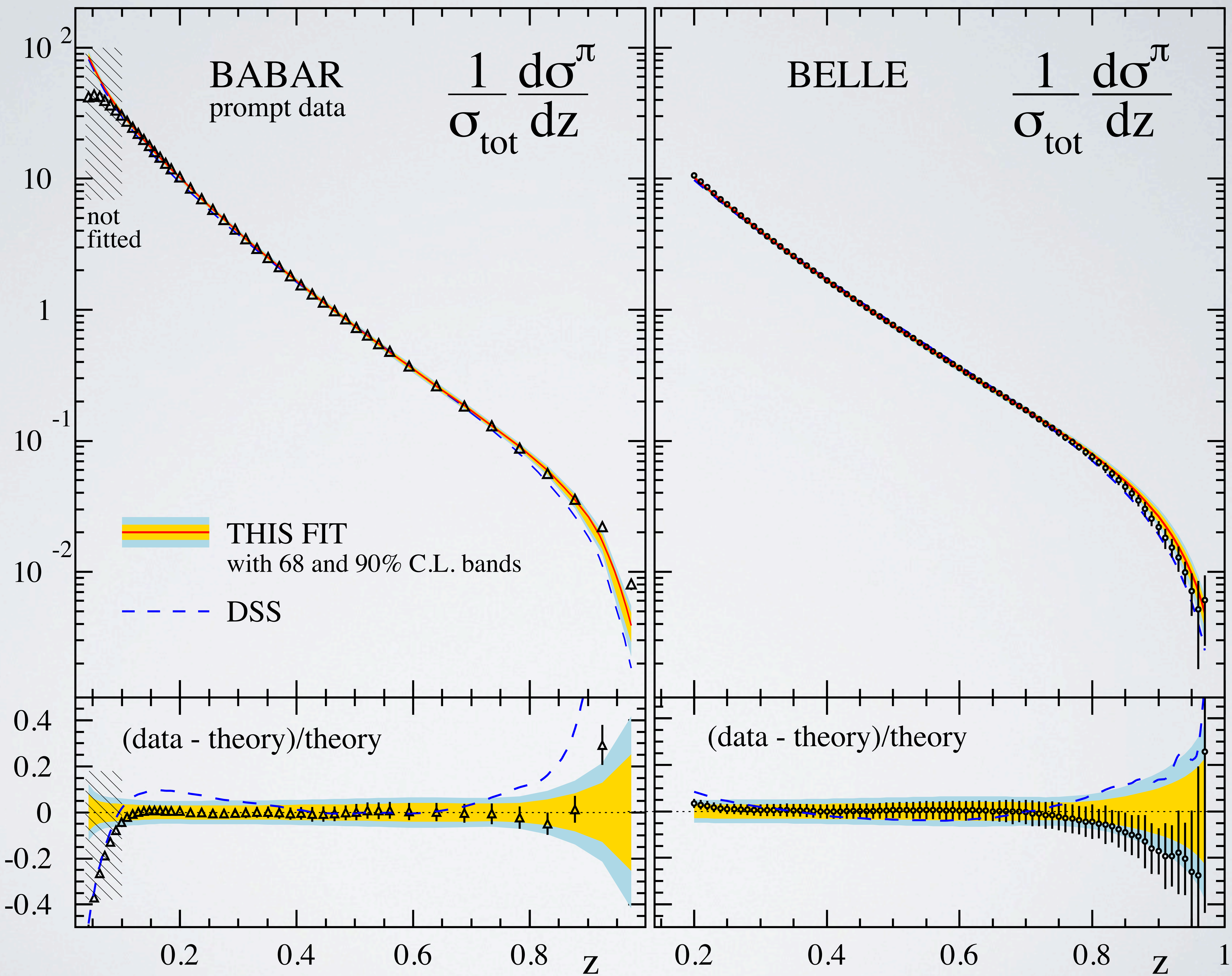


z~0.05-0.2 z>0.2

Q~MZ

very precise up to z~0.8

Q~10 GeV



BABAR
prompt data
$\frac{1}{\sigma_{tot}} \frac{d\sigma^{\pi}}{dz}$
not fitted
THIS FIT
with 68 and 90% C.L. bands
DSS
(data - theory)/theory
BELLE
$\frac{1}{\sigma_{tot}} \frac{d\sigma^{\pi}}{dz}$
(data - theory)/theory
z
z

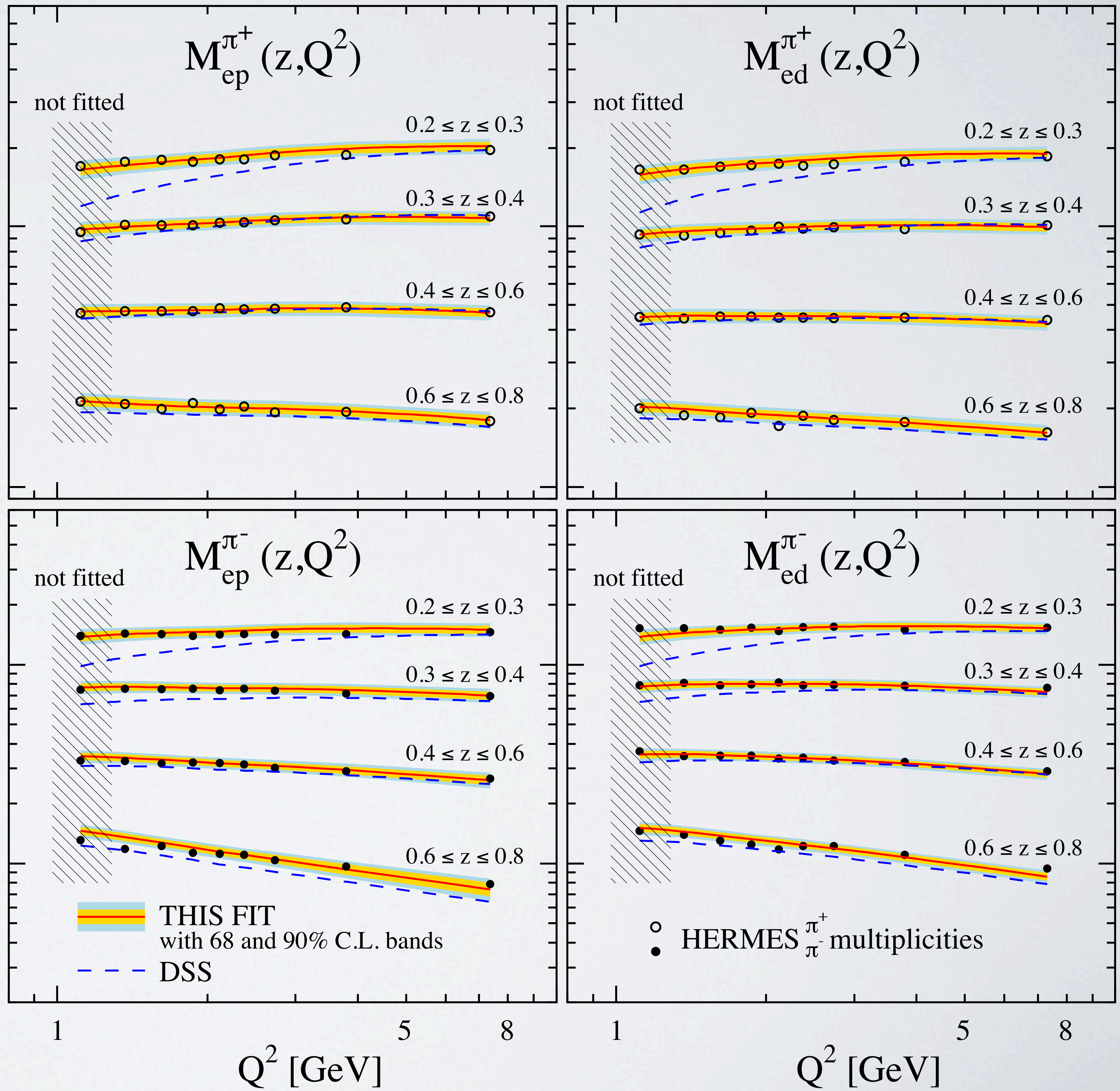
New SIDIS

final HERMES (p & d) new COMPASS

same kinematics

much better agreement (MSTW?)

deuteron~flavor separation



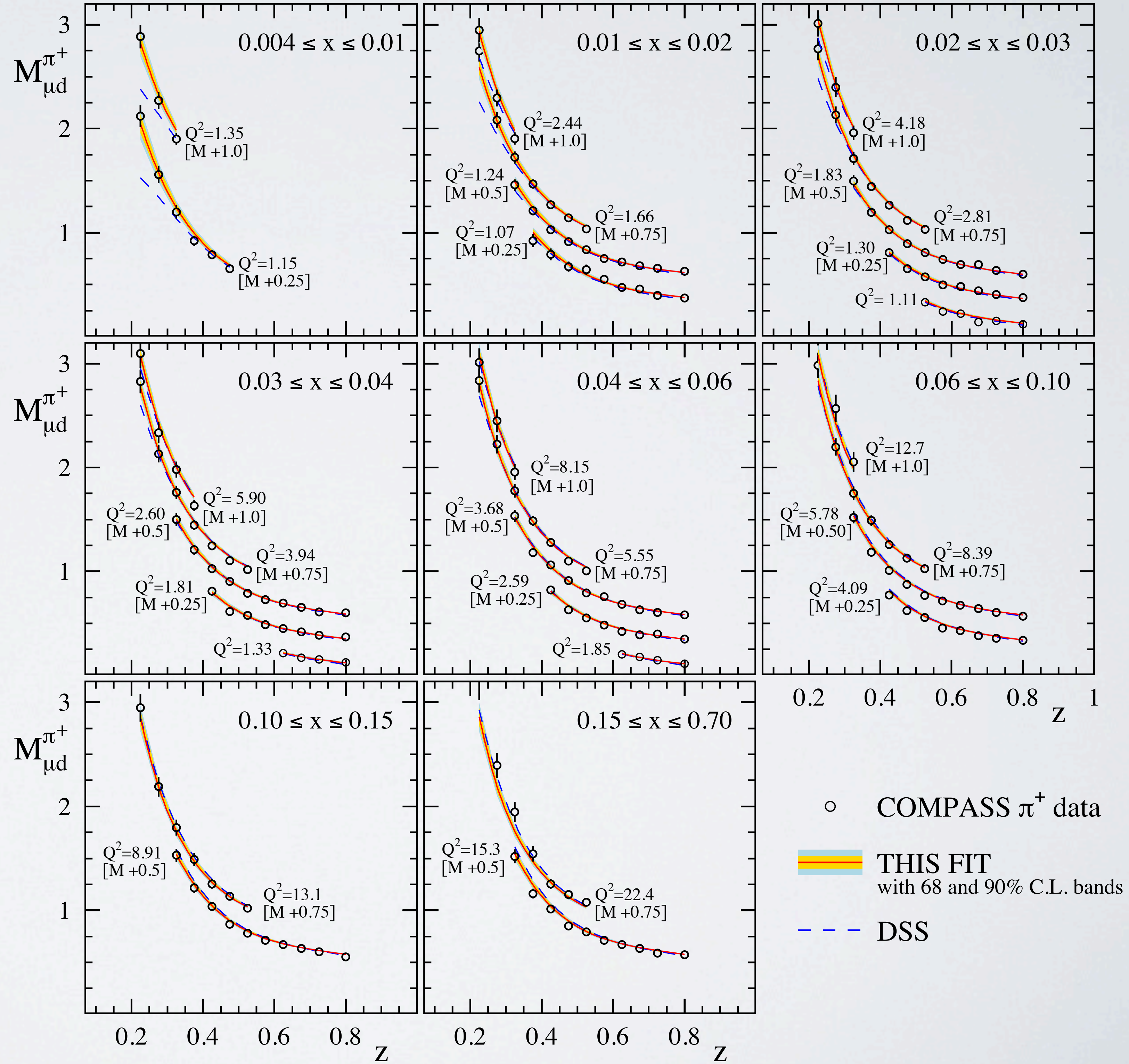
New SIDIS

final HERMES (p & d) new COMPASS

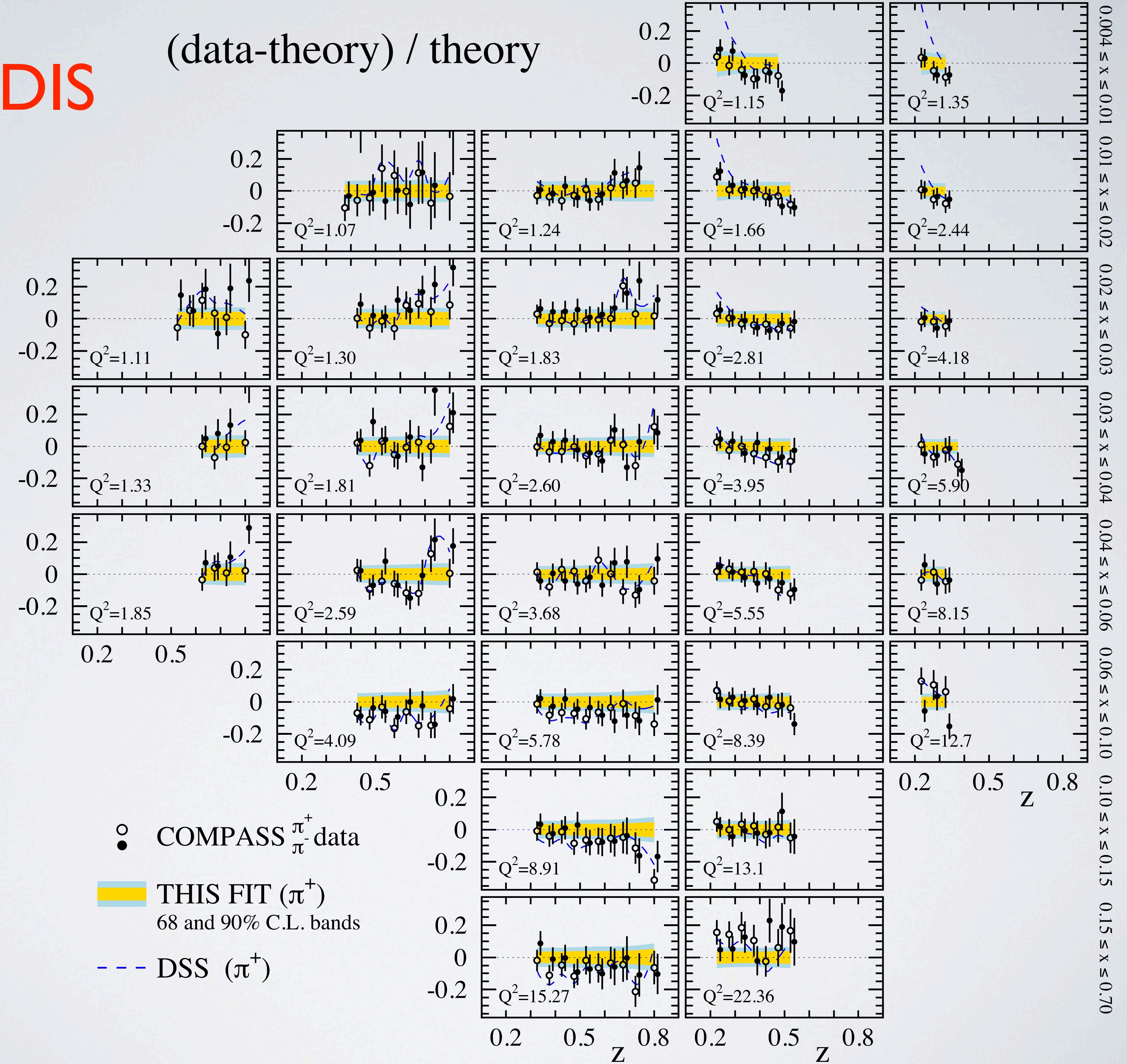
bins in x

continuous z

larger Q^2

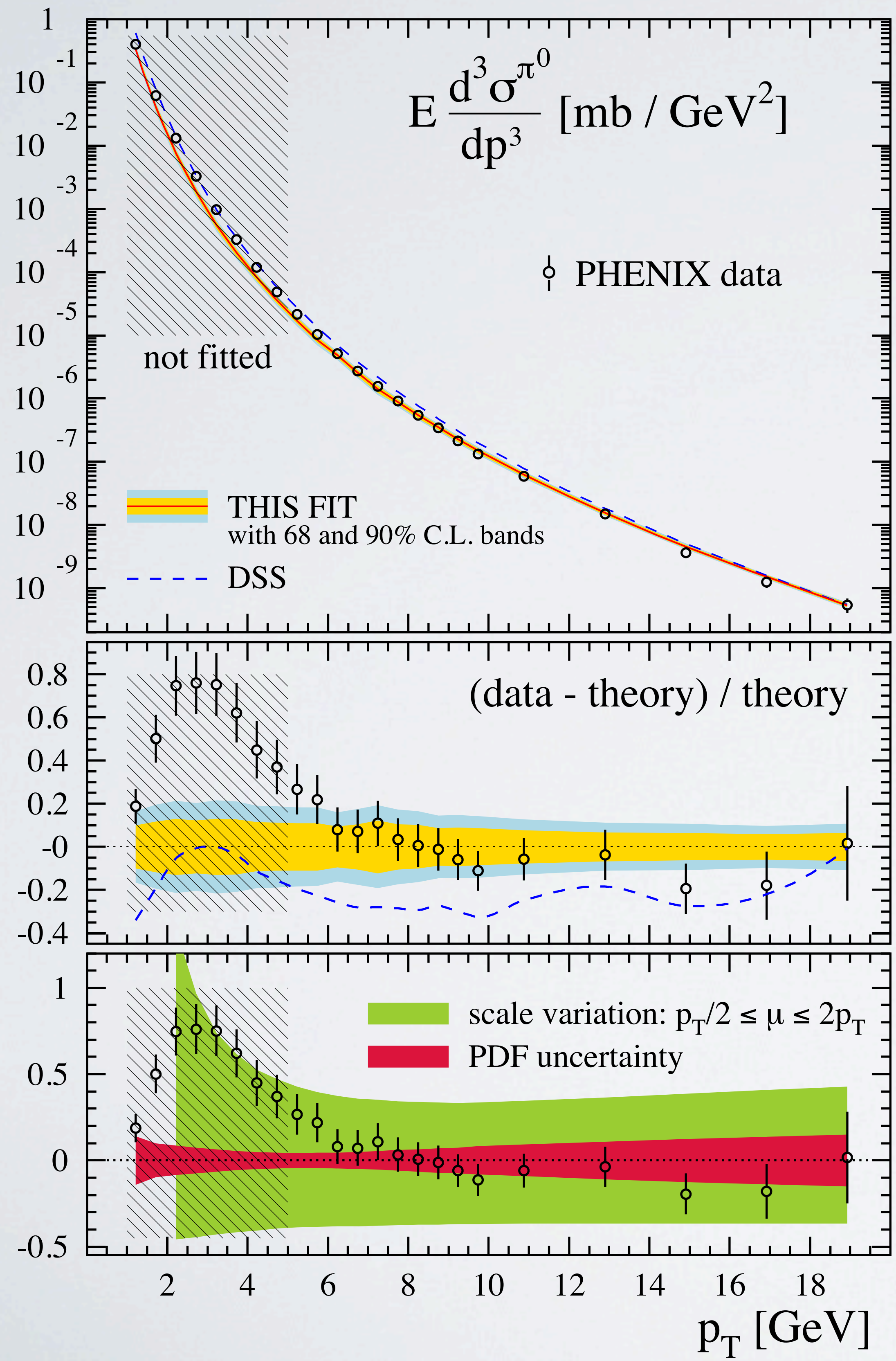


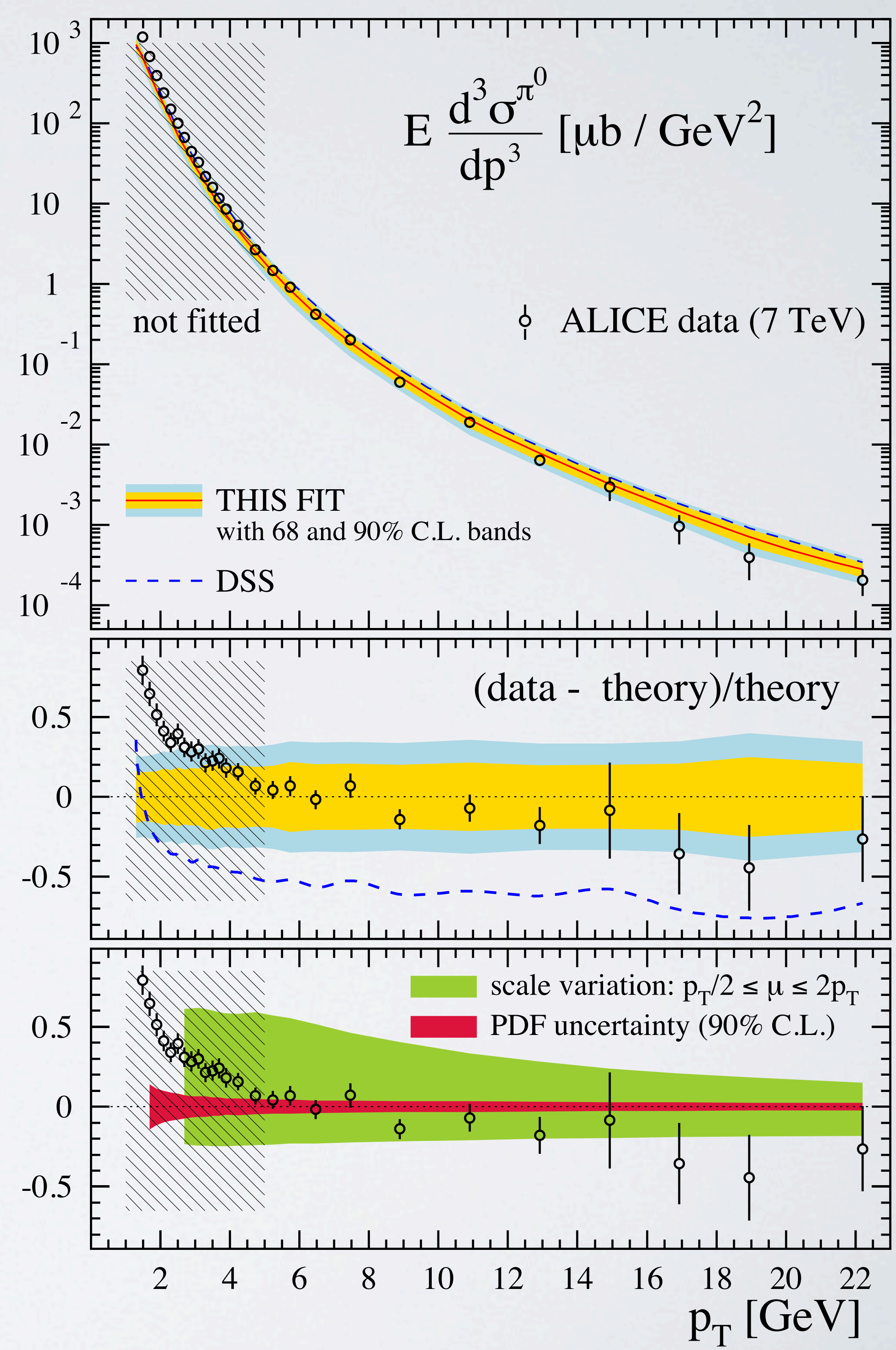
New SIDIS



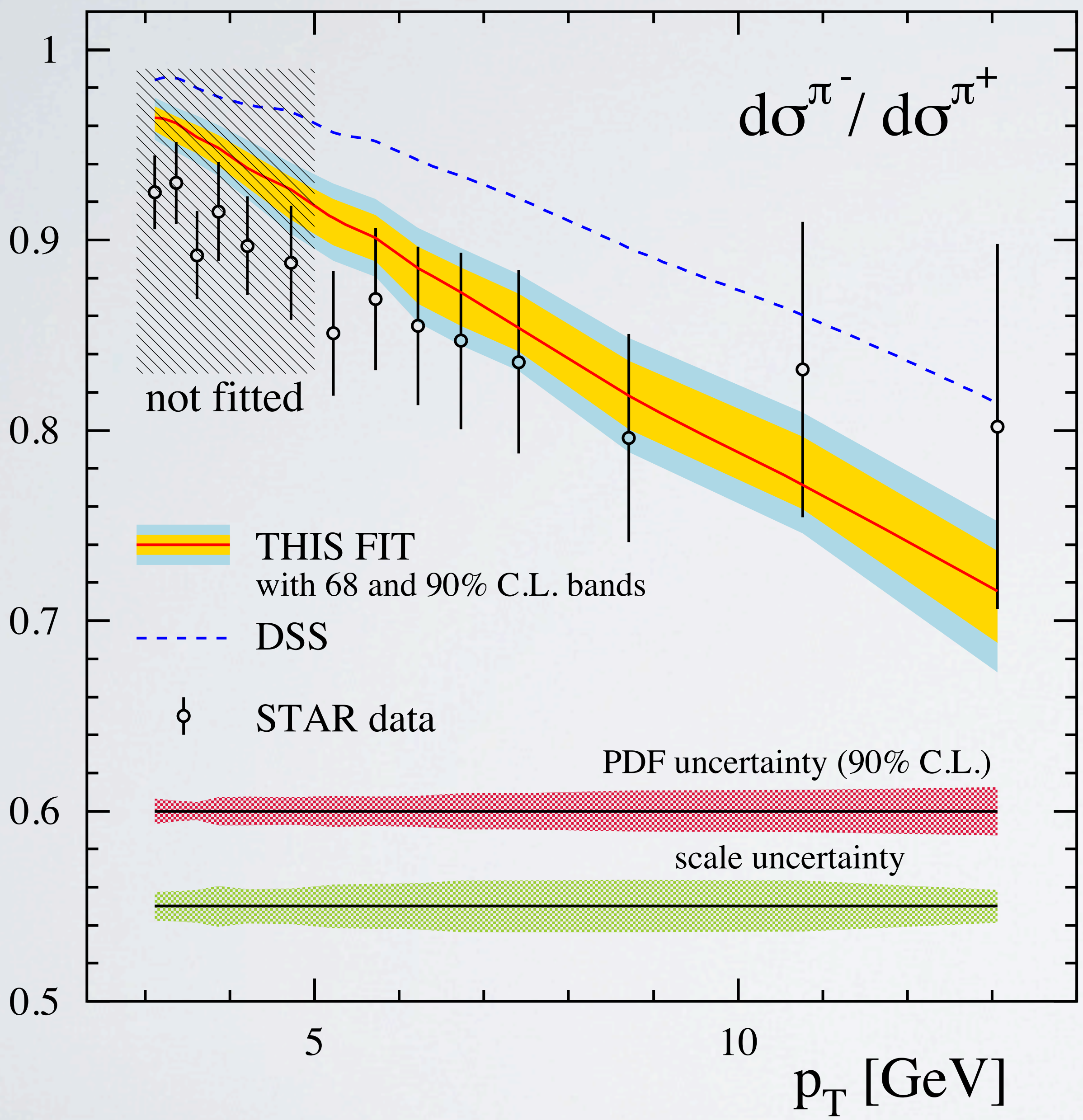
(data-theory) / theory
COMPASS π^+ π^- data
THIS FIT (π^+)
68 and 90% C.L. bands
DSS (π^+)

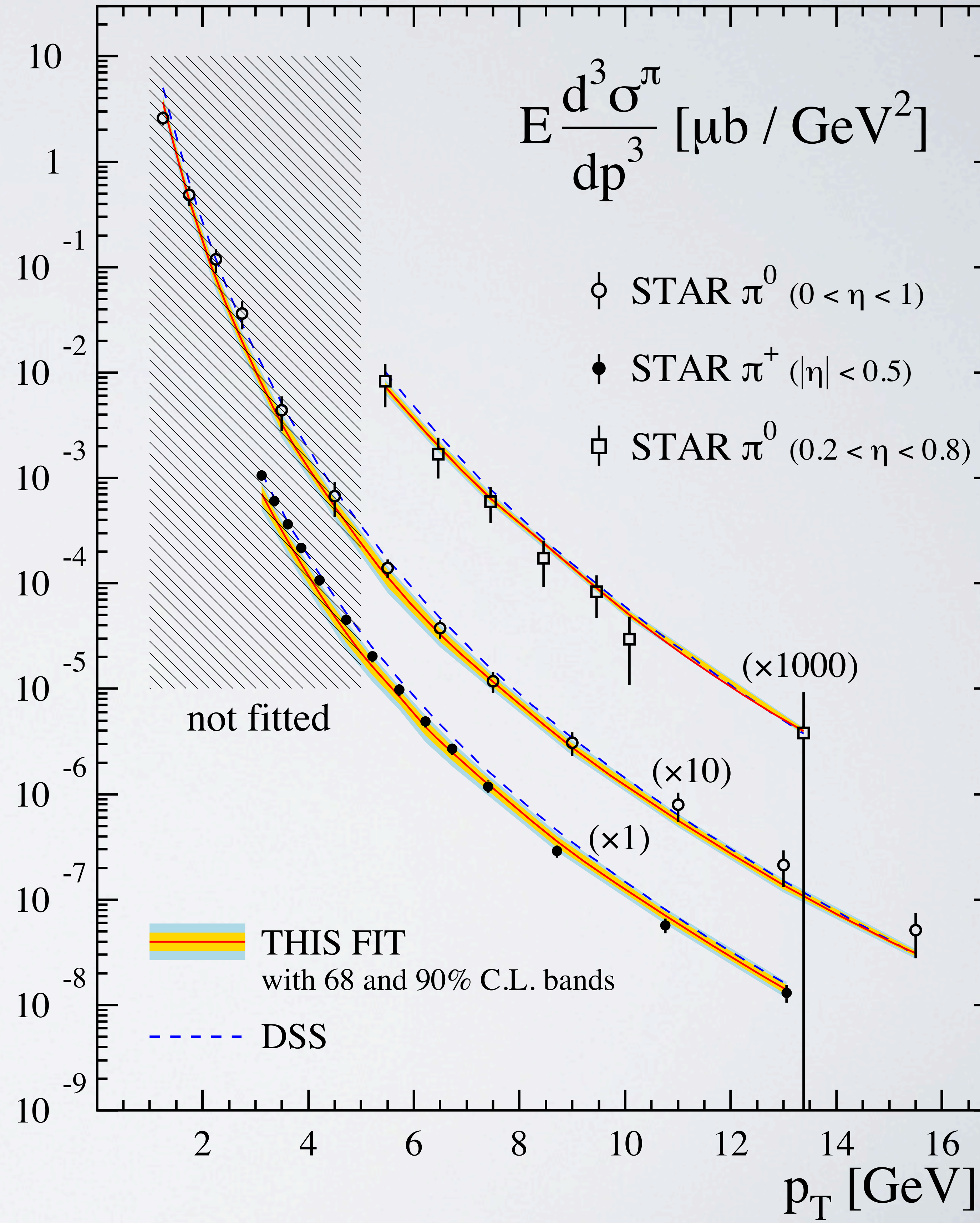
New pp RHIC and LHC



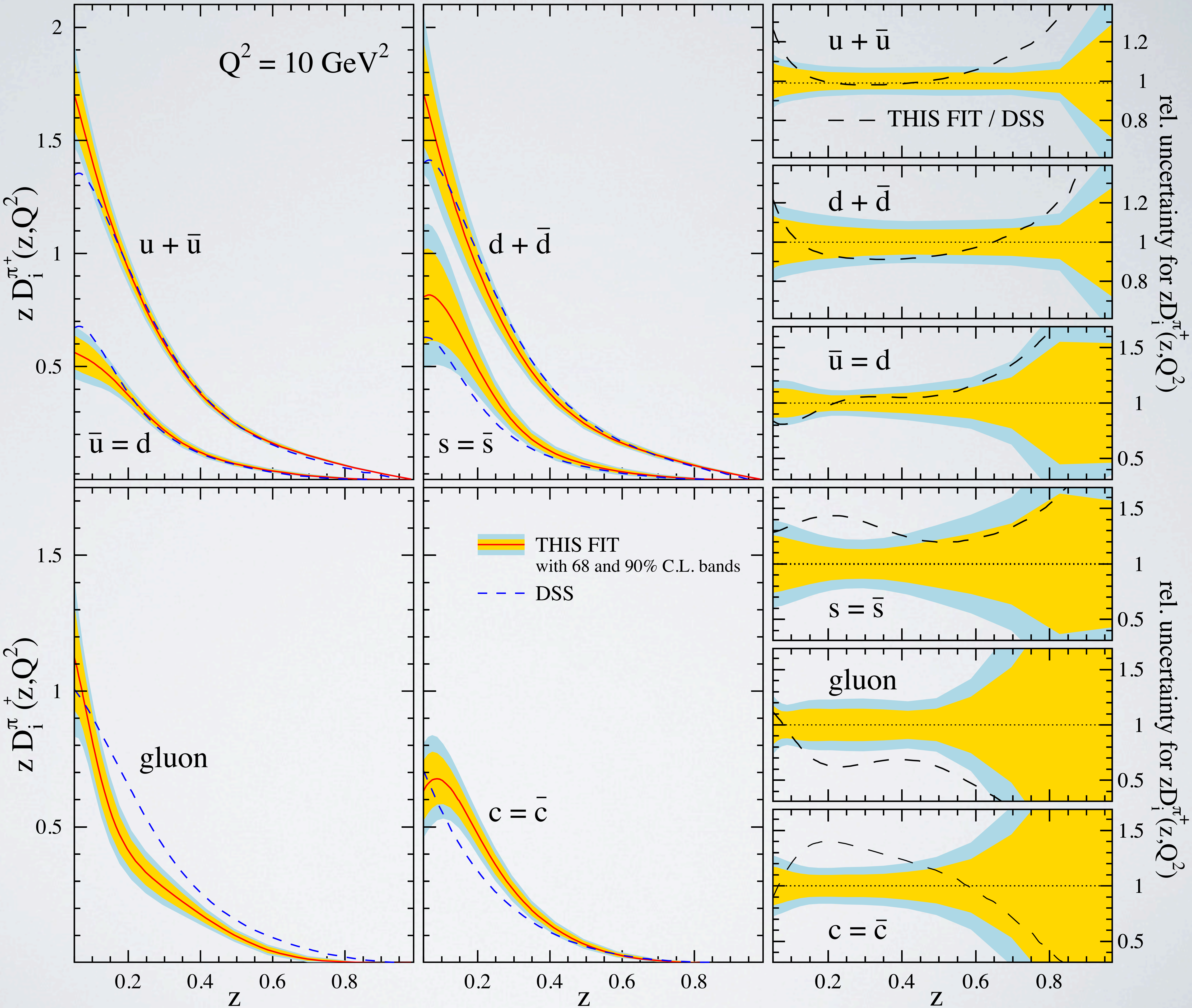


New pp RHIC and LHC





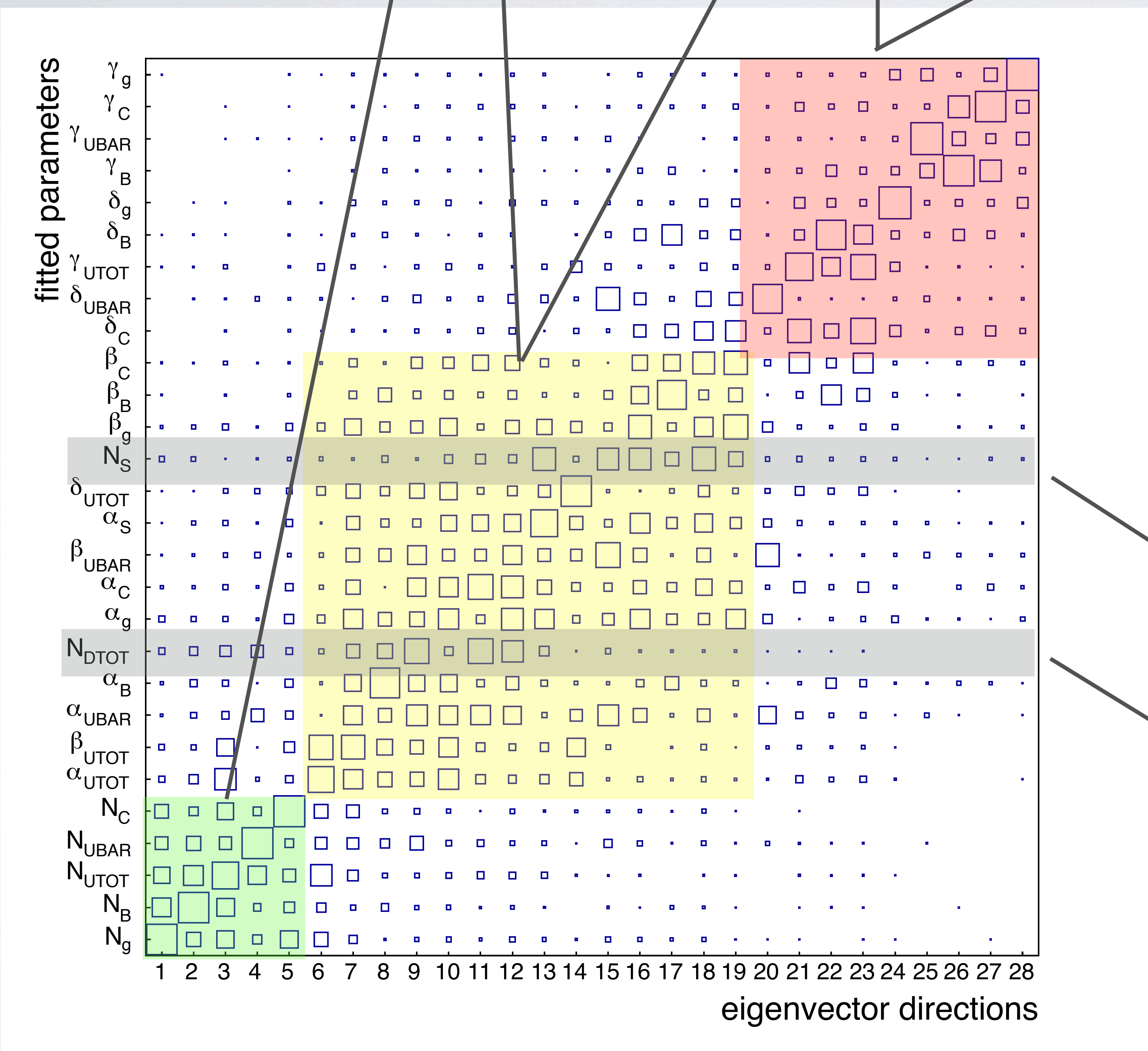
FFs:

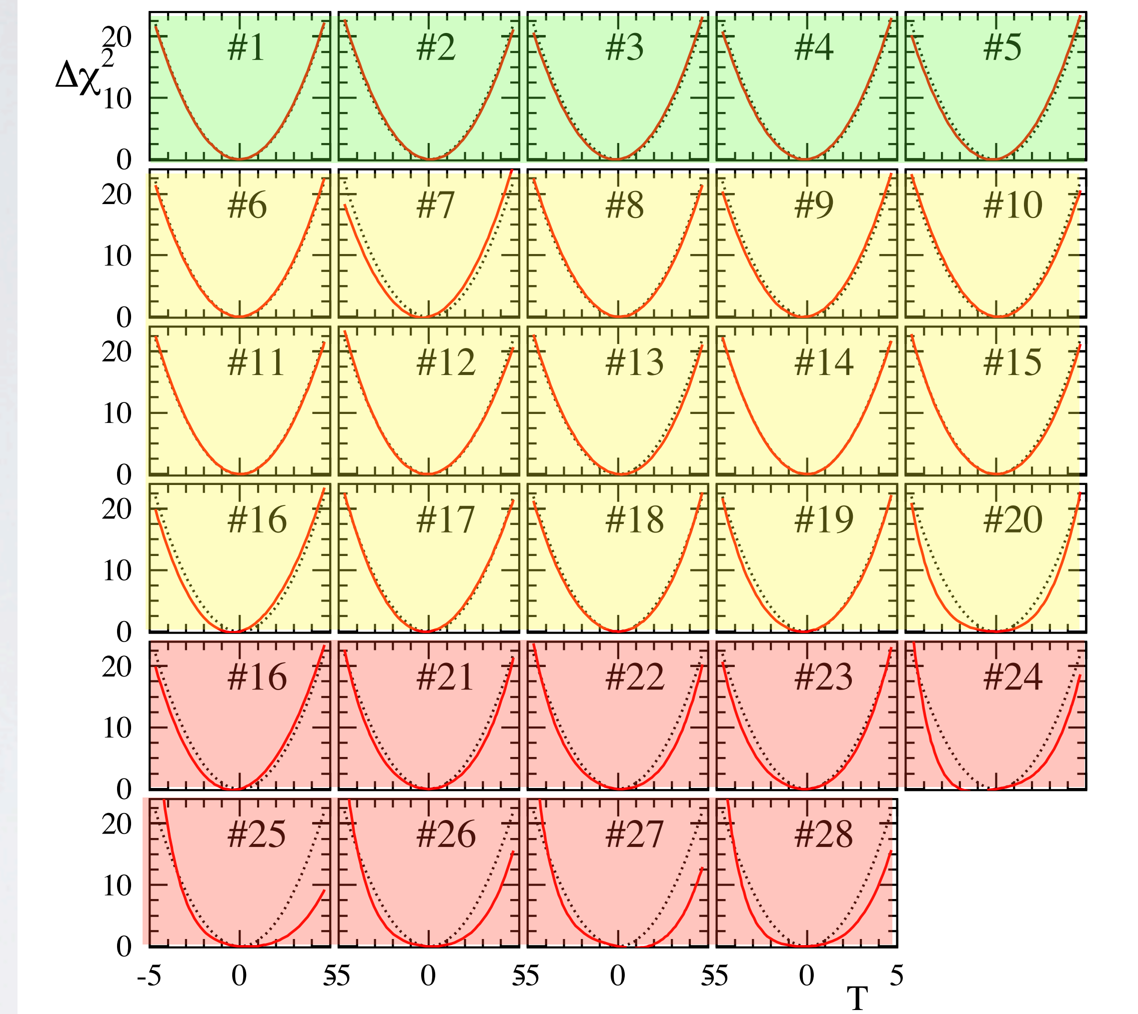


$Q^2 = 10\ GeV^2$
$u + \bar{u}$
$\bar{u} = d$
$d + \bar{d}$
$s = \bar{s}$
gluon
$c = \bar{c}$
THIS FIT
with 68 and 90% C.L. bands
DSS
THIS FIT / DSS
$z\,D_i^{\pi^+}(z,Q^2)$
rel. uncertainty for $zD_i^{\pi^+}(z,Q^2)$
z

Is the Hessian safe?

$$D_i^{\pi^+}(z,Q_0^2) = N_i z^{\alpha_i}\left[(1-z)^{\beta_i} + \gamma_i (1-z)^{\delta_i}\right]$$





SU(3):

$$D_{\bar{s}}^{\pi^+}(z,Q_0^2) = N_s z^{\alpha_s} D_{\bar{u}}^{\pi^+}(z,Q_0^2)$$

SU(2):

$$D_{u+\bar{u}}^{\pi^+}(z,Q_0^2) = N_{d+\bar{d}}\, D_{u+\bar{u}}^{\pi^+}(z,Q_0^2)$$

Old and new numbers

	DSS	DSS-II
GLOBAL	843.7/392 (2.15)	1154.7/953 (1.21)
LEP-SLAC	500.1/260 (1.92)	412.7/260 (1.58)
BELLE-BABAR	-	90.4/117 (0.77)
HERMES	188.2/64 (2.94)	175.0/128 (1.36)
COMPASS	-	403.2/398 (1.01)
RHIC	160.8/68 (2.36)	43.0/38 (1.13)
LHC	-	27.7/12 (2.31)

Outlook:

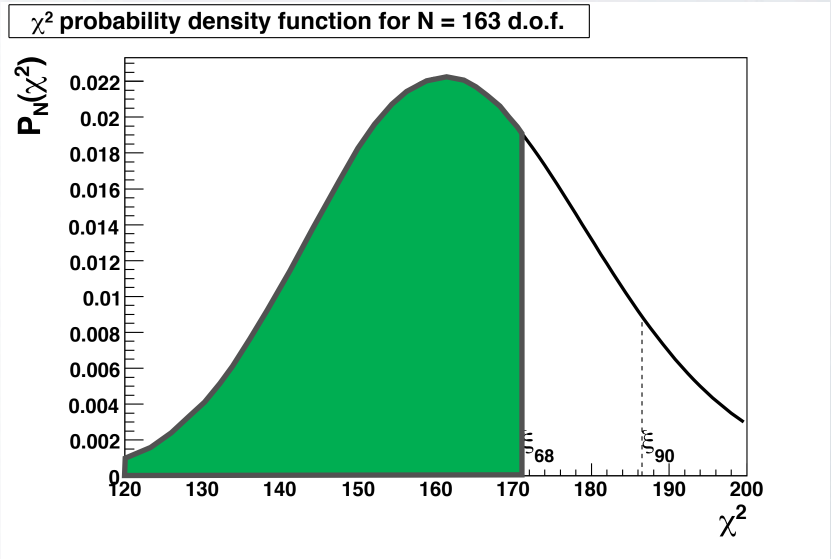
Experiment

SIDIS: Compass K
SIDIS: Hermes K ✔
SIDIS: Zeus K,Λ ✔
SIA: Belle/BaBar K ✔
pp (K,p,$\bar{p}$,η,Λ,$h^{\pm}$) ratios?
EIC (combined PDFs & FFs?)

Theory

SIDIS at NNLO
pp NNLO?
heavy quark masses
Threshold resummation
More/different global fits to compare

$$\int_0^{\xi_{68}} d\chi^2 \, \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2}\Gamma(N/2)} = 0.68$$



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